AN ABSTRACT OF THE DISSERTATION OF

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Title: Mathematical Narratives and Perspectives of Emerging Elementary Educators

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Annette M. Holba, PhD
Dissertation Chair

This narrative inquiry study explored how emerging elementary educators perceived their understanding of mathematics learning and teaching as they became elementary educators. The study considered how past experiences influenced emerging educators' frames of reference as they began to teach elementary mathematics. Methods consisted of individual interviews and focus groups with pre-service and in-service elementary educators. Participants also completed a mathematics self-efficacy inventory. Findings indicated participants embraced new mathematics strategies and valued mathematics reasoning over memorization, however, deeply rooted habits of mind, particularly related to beliefs about mathematics abilities persisted. Participants described challenges primarily associated with mathematics programs used by their schools and districts. Furthermore, many participants discussed a practice disconnect between what they learned in their teacher preparation programs and the realities of being in a classroom. Teacher educators should acknowledge prior experiences and incoming beliefs of pre-service teachers and may need to provide opportunities to question and deconstruct unproductive beliefs and attitudes in the pre-service classroom.

Keywords: teacher education, transformational learning, elementary mathematics

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Mathematical Narratives and Perspectives of Emerging Elementary Educators

By

Stephanie M. Banks

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I understand that my dissertation will become part of the permanent collection of Plymouth State University, Lamson Learning Commons. My signature below authorizes release of my dissertation to any reader upon request.

Stephanie M. Banks

Name of Student, Author

DEDICATION

This dissertation is dedicated to my family. To John, who has kindly listened to my endless dissertation thoughts and ideas, and who has been along for the ride through all its ups and downs. To Ella, who has been my biggest supporter, who has genuinely listened to me practice presenting my work, and who was never shy about offering me input. I hope you always recognize your wisdom and cherish each opportunity life presents you with. To Evie, who has shown me patience and understanding and who inspires me to be the best version of myself every single day.

Lastly, this dissertation is dedicated to all the students who have questioned their mathematical abilities at one point or another. For all those who have said, “I’m not a math person,” I hope you will be inspired to give mathematics another chance and find beauty and enjoyment in mathematics.

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Mathematical Narratives and Perspectives of Emerging Elementary Educators

Stephanie M. Banks, Plymouth State University

Dissertation Defense: February 9, 2023

Executive Summary: February 9, 2023

Introduction: This research examined how elementary pre-service and new in-service teachers perceived their understanding of mathematics learning and teaching as they became elementary educators.

Problem of Practice: As pre-service teachers enter their teacher education programs, they bring with them their own set of personal experiences, beliefs about themselves as mathematics learners, and ideas about mathematics learning and teaching. Many elementary pre-service educators enter their programs with low levels of mathematics self-efficacy (Geist, 2015; Looney et al., 2017) and gaps in their own understanding of mathematics concepts (Fuentes et al., 2014; Van Steenbrugge et al., 2014). The past experiences of pre-service educators are important to consider because these have shaped their perceptions and the lens through which they view mathematics learning and teaching.

Research Method: This qualitative research study utilized a narrative inquiry approach to engage participants in individual interviews and focus groups. One focus group consisted of pre-service elementary educators and the other focus group consisted of in-service elementary teachers in their first three years of classroom teaching. All participants also completed the Self-Efficacy for Teaching Mathematics Inventory (SETMI; Mcgee & Wang, 2014).

Summary of Findings: Although participants embraced new strategies and valued mathematics reasoning over memorization, deeply rooted habits of mind, particularly related to beliefs about mathematics abilities persisted. Additionally, participants described curriculum constraints challenges and expectations primarily associated with mathematics programs used by their schools and districts. Furthermore, many participants discussed a practice disconnect between what they learned in their teacher preparation programs and the realities of being in a classroom.

Limitation(s) of Study: Participant recruitment during the Covid-19 pandemic was a challenge. Additionally, the qualitative nature of the study provides a detailed perspective of narratives and perspectives that may not be readily generalizable to other populations.

Implications/Significance of Study: Teacher educators should acknowledge prior experiences and incoming beliefs of pre-service teachers and may need to provide opportunities to question and deconstruct unproductive beliefs and attitudes in the pre-service classroom.

Chapter One: Problem of Practice

Norah is sitting at the kitchen table with her father staring at her math homework. Norah has been learning the lattice multiplication strategy at school, but her father is unfamiliar with and frustrated by the strategy. Norah's father begins teaching her the "traditional" way to solve multiplication. This creates tension at home over homework. Norah generally views herself as a competent mathematics student and enjoys mathematics. She can readily pick up new concepts during class and has pride in the high marks she has received on her report cards. Norah's school uses a mathematics program and her whole group mathematics lessons are teacher-directed, followed by independent practice in her mathematics workbook. She easily memorizes her facts and procedures yet does not always see the application of the mathematics she is learning. Beginning in middle school, she was placed in an advanced track class. This made Norah feel even more capable as a mathematics learner since she was able to advance a grade level above many of her peers. When Norah reaches Geometry in high school, she begins to face challenges in mathematics. The concepts are not coming as easily, and she is falling behind. Norah starts to feel incapable as her grades are not at a level she generally achieves. She notices other students in the class are grasping concepts quicker and performing better on assignments and assessments. Norah starts to feel like she is no longer a "good" mathematics student. She begins to dread going to Geometry and wonders if she should drop the class to a lower level.

Fast forward a few years, and Norah is in her mathematics methods course within her university's teacher preparation program. She is revisiting some of the

elementary mathematics content she has not seen in years. She starts to notice some misconceptions she had about place value and begins learning new strategies for teaching mathematics that are different from how she learned. She learns best practices and how to address misconceptions. She practices planning mathematics lesson plans and activities.

Norah gets her first job in a third-grade classroom. She is excited and nervous. Her district follows a mathematics program that she is unfamiliar with. She is not sure how to navigate the program and integrate the knowledge and skills she learned from the teacher preparation program. Norah reverts to what she knows about mathematics and mostly follows the program because this is what her school expects.

Introduction to the Problem

This research examined how elementary pre-service and new in-service teachers perceived their development of mathematics learning and teaching as they became elementary educators. This study explored the experiences, relationships, and conditions that pre-service teachers and new in-service teachers viewed as influential in developing their capacities to learn and teach elementary mathematics.

This chapter will deliver background information on this problem of practice and will offer a rationale for the research study. The following section of this chapter will provide an overview and historical perspective of mathematics education in the United States. It will introduce the reader to information on social, economic, and political influences that have ultimately had an impact on teacher education programs. The next section will discuss the local context and supplies the reader with an understanding of not only local teacher education programs, but information

concerning the local community and K-12 school districts in a New England state. Next, this problem of practice will be considered from an equity lens and the importance of addressing the problem will be discussed. Lastly, this chapter will identify the specific focus of the research study and the central research questions.

Global and Historical Perspectives

Mathematics education is not a new issue or concern in the United States, yet after decades of studies, policy actions, and curricular changes, mathematics education and achievement remain prevalent focal points in the educational agenda. Permuth and Dalzell (2013) argued "methodology for teaching mathematics responds to the directions of change, economic pressure, and scientific and nonscientific progress because mathematics is central to a nation's standing and power" (p. 236) and pointed to critical social and political influences that have shaped mathematics education over the span of many decades. Beginning in the 1950s with the launch of Sputnik and concerns that Americans were losing the Space Race, education, in particular mathematics and the sciences, came under public scrutiny (Permuth & Dalzell, 2013). In the late 1960s, the First International Mathematics Study illuminated a discrepancy between the mathematics achievement of students in the United States compared to their international counterparts (Dossey et al., 2016). Then the 1980s brought significant public attention and increased pressure to improve mathematics education with the publication of *A Nation at Risk*, which painted a grave picture of American education in the early 1980s that demanded change and precipitated the creation of standard recommendations from organizations including the National Council for Teachers of Mathematics (NCTM) (Dossey et al., 2016). Over the course of the next

several decades, standards and recommendations were revised and implemented across the country. States began to align their curriculum, instruction, and assessment with NCTM standards (Dossey et al., 2016). These recommendations and standards not only influenced K-12 schools and in-service teachers; they also impacted pre-service teachers and teacher education programs through NCTM's work with accrediting organizations (Dossey et al., 2016). With the turn of the century, mathematics education continued to be influenced, most visibly through key legislative actions.

In 2001 George W. Bush enacted No Child Left Behind (NCLB), which consisted of school accountability measures and standardized testing (Permuth & Dalzell, 2013). The original intent of NCLB was to close the equity gap observed across the nation; however, it created friction between the federal government and states, as penalties were issued for those schools not meeting adequate yearly progress (Neill et al., 2004). Critics claimed the law encouraged educators to teach to standardized tests, was underfunded, and set the neediest school districts up for failure (Neill et al., 2004). NCLB was replaced in 2015 with the Every Student Succeeds Act, which allowed states more flexibility to set standards and accountability measures (Dossey et al., 2016). However, by this time another initiative was already causing controversy.

At the start of the new decade in 2010, the Common Core State Standards in Mathematics (CCSSM) were introduced with the goal of shared standards for what students should master at various educational levels (Dossey et al., 2016). Dossey et al. (2016) described CCSSM's intended goal to:

> ensure that students understand mathematics as structure, reasoning, and procedures that make sense (rather than a long list of rules to follow) and that they acquire the ability to reason with and use mathematics. As such, CCSSM was envisioned as a set of standards that would be adopted or adapted by the states, allowing all students to have equal access to similar instructional activities based on materials created to reflect a similar vision of school mathematics. (p. 20)

The release of CCSSM initiated several influential documents and recommendations, including The Conference Board of Mathematical Sciences' 2012 report, Mathematical *Education of Teachers (MET) II* (Dossey et al., 2016).

MET II emphasizes the importance of pre-service educators' mathematics preparation at all levels and outlines several key considerations and recommendations (Conference Board of Mathematical Sciences, 2012). The report highlights the significance of pre-service educators developing mathematical knowledge for teaching (Conference Board of Mathematical Sciences, 2012). This knowledge is divergent from knowledge of mathematical concepts or procedural knowledge needed to carry out or perform mathematics. Mathematical knowledge for teaching is specialized knowledge that is relevant to teaching mathematics and requires a profound understanding of the subject and how students are interacting with concepts and ideas (Conference Board of Mathematical Sciences, 2012).

MET II argues that neither a pre-service educator's prior experiences as a mathematics student nor coursework in advanced mathematical concepts are sufficient in preparing the individual to teach mathematics. Rather future educators need courses

and experiences that "aim to remedy weaknesses in mathematical knowledge, but also help teachers develop a deeper and more comprehensive view and understanding of the mathematics they will or already do teach" (Conference Board of Mathematical Sciences, 2012, p. 23). Furthermore, MET II recommends elementary teacher preparation programs have at least 12 semester hours of coursework on studying mathematics aligned with CCSSM (Conference Board of Mathematical Sciences, 2012). This includes coursework that examines how concepts across the elementary grade bands build upon each other and prepare students for mathematics they will learn in middle school (Conference Board of Mathematical Sciences, 2012). Additionally, MET II calls on mathematicians and math departments to take an active and collaborative role in teacher preparation programs and coursework, including those that prepare elementary educators. Decades of efforts, initiatives, and research have influenced mathematics education and informed how teacher preparation programs prepare the next generation of teachers.

Mathematics education has been molded and adapted over time and this evolution influences pre-service teachers in several ways. Their own educational histories in mathematics have been shaped by these aforementioned political, social, and economic factors. They may have been students in classrooms where their own teachers were grappling with how to change their curriculum, teaching practices, or assessment measures. Their own mathematics experiences have an impact on their incoming knowledge of mathematics and teaching. There are further considerations specifically for prospective elementary pre-service teachers.

A detrimental but widely common viewpoint in the United States is that elementary mathematics is simple or basic, yet this simplistic view is flawed. Ma (1999) argued, “elementary mathematics is not superficial at all, and anyone who teaches it has to study it hard in order to understand it in a comprehensive way” (p. 146). As noted earlier, teachers require mathematical knowledge that goes beyond correctly solving algorithms or even having a sound conceptual understanding (Conference Board of Mathematical Science, 2012). Teachers need mathematical knowledge that is unique to the profession; they need to have mathematical knowledge for teaching. Common Core State Standards have the intent to develop student understanding, but to fully capitalize on the standards, educators need to be well-versed in concepts and be equipped to provide opportunities for exploration and critical thinking around a given topic (Griffin & Ward, 2014). Griffin and Ward (2014) indicated the importance of pre-service educators’ understanding of concepts to address questions and misconceptions as they arise in the classroom. This requires educators to not only be deeply knowledgeable of mathematics topics and progressions but also able to understand and respond to student thinking.

This specialized knowledge is critically important for elementary educators who teach mathematics, and it is far from superficial or basic. Elementary educators in grades pre-kindergarten through fifth grade play an important role in students’ growth and understanding of mathematics that may influence their future trajectory. The elementary classroom provides the base of this mathematics knowledge and research suggests early mathematics success is a predictor of later academic success in mathematics (Rittle-Johnson, 2017). Elementary educators help set the foundation of

students' mathematical understanding, making it crucial to prepare pre-service elementary educators to teach mathematics comprehensively and effectively.

However, there are several challenges to ensuring pre-service educators are well-prepared to teach their future students mathematics. Elementary teachers are generalists, meaning they often teach multiple subjects with mathematics being one of many subjects they must be prepared to teach. Additionally, most elementary educators and prospective elementary educators do not have extensive backgrounds or advanced knowledge of mathematics (Schmidt et al., 2017). Furthermore, many elementary educators have had negative experiences associated with mathematics from their own educational histories, which can influence their confidence and ability to teach concepts with depth (Bekedemir, 2010; Geist, 2015). This means that pre-service teachers may be entering their teacher education programs with limited or inaccurate knowledge of mathematics, have negative attitudes towards mathematics, possess counterproductive beliefs about the nature of mathematics, and need to develop knowledge and proficiency to teach a wide variety of subjects in addition to mathematics.

As pre-service teachers enter their teacher education programs, they bring with them their own set of personal experiences, beliefs about themselves as mathematics learners, and ideas about mathematics learning and teaching. The past experiences of pre-service educators are important to consider because these have shaped their perceptions and the lens through which they view mathematics learning and teaching. Research has indicated that many elementary pre-service educators enter their programs with low levels of mathematics self-efficacy (Geist, 2015; Looney et al.,

2017). Furthermore, many pre-service elementary educators enter their programs with gaps in their own understanding of mathematics concepts (Fuentes et al., 2014; Van Steenbrugge et al., 2014). As prospective elementary teachers enter their teacher education programs, it becomes important to consider their incoming knowledge and beliefs and consider how these prior experiences influence educators as they engage in their teacher preparation programs and classroom teaching positions.

Local Contexts

Education

This study was situated within a small state in the northeast. Before looking at the collective and individual teacher preparation programs within the local contexts of this state, it is noteworthy to consider the local educational landscape. Not all individuals who complete a local teacher preparation program remain in the state to teach, but many do. It, therefore, becomes important to consider these pre-service teachers' future teaching environments. Additionally, many local teacher preparation programs include field experiences within the local school districts. The local communities may not only become future employment sites, but they may also be influential environments where pre-service teachers develop and practice teaching skills. This section and the next will consider state demographic data in addition to mathematics achievement data in an effort to consider teacher preparation programs not as isolated academic programs, but as programs situated and influenced by the local school communities.

During the 2018-2019 school year, there were 177, 398 students enrolled in schools throughout the state (New Hampshire Department of Education [NHDOE], 2019). According to the NHDOE (2019), 84.95% of the state's student population is

White, and 28.25% of students are considered “economically disadvantaged”. However, the diversity and socioeconomic differences are variable depending on the local community. This becomes an important factor to consider because teacher preparation programs are located in different areas of the state, and even within a program through different partnerships, pre-service teachers may have very different field experiences.

Mathematics Achievement

In examining and comparing national academic performance indicators, this New England state ranks close to the top. According to data from the National Assessment of Educational Progress (NAEP), fourth graders in the state scored significantly higher than the national average (The Nation’s Report Card, 2019). Data from 2019 indicate that 86% of fourth graders in the state are at or above a basic level in mathematics and 46% of fourth graders in the state are at or above proficient (The Nation’s Report Card, 2019). By eighth grade, 77% of eighth graders in the state are at or above a basic level in mathematics and 38% of eighth graders in the state are at or above proficient (The Nation’s Report Card, 2019). This New England state is significantly higher than the national average in mathematics at each grade level measure (The Nation’s Report Card, 2019). However, as the data indicate, being at the top does not necessarily mean students in the state are proficient in mathematics. Less than half of fourth graders and even fewer eighth graders are meeting this benchmark. Although this New England state is outperforming most of the nation, there is still a great deal of progress that must be made to move the majority of students toward proficiency in mathematics.

Furthermore, the 2019 mathematics data in the state highlight gaps in scores for particular student groups that are worthy of attention. Fourth-grade mathematics data indicate Black students' average score was 18 points lower than that of White students, and Hispanic students' average score was 19 points lower than that of White students (The Nation's Report Card, 2019). Male students outscored female students by five points and students eligible for Free and Reduced Lunch scored 19 points lower than students who are not eligible for the program (The Nation's Report Card, 2019). These gaps are concerning and indicate that more action needs to be taken to provide equitable access to mathematics education to marginalized groups. Not only do these statistics warrant attention, but they can be used to inform teacher education programs. If certain experiences and aspects within pre-service coursework can be identified by pre-service teachers as important in building depth of mathematical knowledge and mathematics self-efficacy, these can be leveraged to better situate pre-service teachers to provide high-quality mathematics instruction to students throughout the state.

Teacher Education Programs in the Local Context

Although some educators within the state may become licensed through alternative pathways including site-based licensing, out-of-state approval, and demonstrated competencies, many prospective educators complete an approved educator preparation program. For the purposes of this discussion and research, unless otherwise stated, the terms pre-service teacher and pre-service educator will refer to individuals who are enrolled in an approved educator preparation program in a post-secondary setting. The Bureau of Educator Preparation oversees the approval and

review of teacher preparation programs. There are 11 different institutions of higher education in the state that offer approved teacher preparation programs for prospective elementary educators (NHDOE, 2020). These institutions of higher education consist of both public and private institutions, and each have unique characteristics to their programs. Each institution of higher education within the northeastern state serves a different population of pre-service educators, and many of these future teachers become active within the context of local school communities. Pre-service elementary participants of this research study were recruited from the four public institutions within a single New England state.

This research is mindful that each institution of higher education is different in the population it serves, its mission, vision, and the academic and social experiences its students have available to them. Beyond the global culture of the institution, each teacher preparation program has its own exclusive components. Course offerings, faculty expertise, and field experiences within the local community create different experiences for pre-service teachers at each institution.

Advocacy and Ethics

In examining mathematics education evolution and reform over the decades, it is evident that this issue is not isolated solely within curricular or professional training and development. Changes to mathematics education and teacher education have been shaped by social, economic, and political influences (Permuth & Darzell, 2013). Efforts to secure and maintain international competitiveness have fueled a culture of academic achievement. Even as legislation and recommendations seek to address inequities in education for marginalized populations, an equity gap remains (Dossey et

al., 2016). Legislative and curricular changes have not dismantled the perpetual and systemic inequities in access to quality mathematics education. These gaps will likely persist without a conscious effort to address them.

The National Council for Teachers of Mathematics (NCTM; 2014) has identified and prioritized equity within its *Principles and Standards for School Mathematics*, in which it states, "all students [should] have access to high-quality mathematics curriculum, effective teaching and learning, high expectations, and the support and resources needed to maximize their learning potential." Teacher education programs have the opportunity to bring awareness to these issues and prepare future educators to have informed practices and the ability to advocate for their future students. This may require an examination of pre-service teachers' beliefs about mathematics and mathematics learning. If pre-service educators view mathematics ability as a fixed trait, they may view mathematics from a deficit perspective rather than approaching mathematics as accessible for all learners (Dossey et al., 2016).

This view of limited knowledge and ability disproportionately targets marginalized groups (Boaler, 2016; National Council of Supervisors of Mathematics [NCSM] & TODOS: Mathematics for ALL [TODOS], n.d.). The outcomes of these perceptions can impact the future trajectory of students. Mathematics achievement has been used as a gatekeeper to access further academic and career opportunities (NCSM & TODOS, n.d.). These pathways for students begin to be paved at the elementary level and continue to become cemented as students progress into secondary schools. Ability grouping and tracking perpetuates ideas of fixed abilities and provides privileged groups with more resources, high-level teaching tasks, and access to paths

to pursue further STEM learning (NCSM & TODOS, n.d.). Consequently, ability grouping and tracking limits access and resources to marginalized groups.

Boaler (2016) also argued against tracking and ability grouping and suggested instead the use of "equitable strategies" that make mathematics accessible to all students and improve outcomes for students traditionally marginalized (p. 102). These strategies include providing all students with access to "high-level content," promoting growth mindset beliefs about mathematics abilities, encouraging collaborative and inquiry-based learning environments, and reconsidering homework (Boaler, 2016, p. 110). Enacting these equitable practices in the elementary mathematics classroom requires intention and knowledge. As pre-service teachers are engaged in teacher preparation programs, there is an opportunity to discuss and apply these principles and strategies to create an inclusive environment. To provide an equitable experience for their future students, pre-service teachers need a solid depth of knowledge in mathematics and mathematics teaching, and they need to believe in their own capacities to do and teach mathematics, as well as the potential of each of their future students.

Focused Problem of Practice

The knowledge needed to effectively teach elementary mathematics and meet the recommended standards and principles is profound. Emerging professionals in elementary education need to develop a depth and range of mathematical proficiencies themselves, but they also need specialized mathematical knowledge unique to teachers (Ball, 2005; Conference Board of Mathematical Sciences, 2012). Many pre-service teachers come into their teacher preparation programs with a limited understanding of

mathematical concepts or have gaps and misconceptions in their thinking (Fuentes et al., 2014; Van Steenbrugge et al., 2014). Pre-service teachers may also have counterproductive beliefs about the nature of mathematics as a subject or of mathematics learning (Conference Board of Mathematical Sciences, 2012). Research has also indicated pre-service elementary teachers lack mathematics self-efficacy, which may impede their own learning and capacity to develop the needed knowledge to teach effectively (Geist, 2015; Looney et al., 2017). Both limited mathematics knowledge and the lack of self-efficacy can have consequences for educators' students. Therefore, it is crucial to understand experiences, relationships, and conditions that shape emerging elementary educators' beliefs and build their knowledge of mathematics.

The focus of this study was to further contribute to the research on mathematics teaching preparedness and learning experiences of pre-service elementary teachers within their teacher education programs. The purpose of this study was to examine how pre-service and new in-service elementary teachers understand their own capacities to do and teach elementary mathematics. This study sought to understand what pre-service and new in-service teachers identify as factors that influenced their development as novice educators of elementary mathematics. This research focused on the experiences and beliefs of the participants and sought to determine how these perspectives and beliefs influenced their mathematics learning and self-efficacy. Specifically, this research explored how pre-service and new in-service elementary teachers understood their own evolving capacity to do and teach mathematics by engaging participants in focus groups and individual interviews.

Central Research Question

1) How do emerging elementary teachers perceive the experiences and conditions, and relationships that influence their development as learners and teachers of mathematics?

Chapter Two: Review of Knowledge for Action

This research examined how elementary pre-service and new in-service teachers perceived their development of mathematics learning and teaching as they became elementary educators. This study explored the experiences, relationships, and conditions that pre-service teachers and new in-service teachers viewed as influential in developing their capacities to learn and teach elementary mathematics. The terms "pre-service educator" and "pre-service teacher" are used to describe individuals who are enrolled in a teacher preparation program with the goal of becoming elementary teachers. Although alternative routes to teacher certification exist, the emphasis of this study was on pre-service teachers who are enrolled in an approved teacher education program at an institution of higher education and those who have completed an approved teacher education program. This study focused on pre-service programs at public institutions of higher education and new in-service teachers employed by elementary schools in a northeastern state. This literature review focuses on research that has resulted from these continued efforts, with a particular emphasis on the knowledge needed by elementary educators to teach mathematics and the influential role of mathematics self-efficacy and mathematics teaching self-efficacy. Chapter Two will reorient the reader to the stated problem of practice and provide an examination of relevant theories and empirical studies. To understand the complexity of this problem of practice it is necessary to build a theoretical framework that encompasses theories relevant to mathematics learning and teaching, adult learning theories, and self-efficacy theory. The first part of this chapter will examine these relevant theories. The next section of this chapter will analyze related empirical

studies, focusing on mathematical knowledge for teaching, mathematics self-efficacy, beliefs, and confidence; and opportunities to learn within teacher education. The next section will discuss the local characteristics and context of this problem with New Hampshire, and the final section will provide a summary of what has been done to address the problem of practice, as well as argue the need and significance of this study.

Efforts have been made over the past several decades to improve mathematics achievement within the United States and attention has shifted toward improving teacher education programs (Ball et al., 2005). As a starting point, it is important to recognize there is no one shared or standard curriculum that guides teacher education programs in preparing future elementary teachers to effectively teach mathematics in the United States. There is limited consistency even within programs and across the teacher education field (Ball et al., 2009). Ball et al. (2009) argued for a common experience to teach future educators the specialized skills for teaching mathematics. This call urged teacher educators to identify and then utilize "high leverage practices," or those used frequently and that have powerful implications for teaching effectiveness (Ball et al., 2009, p. 460; Grossman et al., 2009). It is not realistic within teacher education programs to teach every needed skill or mathematical idea a teacher will need throughout their mathematical teaching career, yet strategic choices of high-leverage practices, such as leading a mathematics discussion or explaining and modeling content, can maximize exposure to effective teaching strategies. Teacher educators can expose pre-service teachers to the most potent instructional activities

that develop mathematical knowledge for teaching during the teacher education program (Ball et al., 2009).

Grossman et al. (2009) argued teacher education programs should focus on engaging pre-service teachers in core practices that are unique to teaching, which includes skills for enacting instructional activities. In many instances, there is a divide between foundation and method courses. This creates a disconnect between theory and practice, which Grossman et al. (2009) suggested should be integrated. Grossman et al. (2009) claimed that by emphasizing core practices within teacher education programs, these programs will begin to bridge the divide between theory and practice. Core practices have the potential to impact future educators' student achievement outcomes and can include learning about student understanding and learning to lead discussions (Grossman et al., 2009). Teacher education programs provide a safe environment for pre-service educators to develop skills, try strategies, and receive on-the-spot feedback on the use of core practices. Grossman et al. (2009) introduced the term "pedagogies of enactment" (p. 283), which envisions teacher education as "laboratory-like settings" (p. 284). In this type of environment, "approximations of practice" allow for simulations and opportunities to practice and rehearse scenarios (Grossman et al., 2009, p. 283).

Ball et al. (2009) created a set of criteria to choose high-leverage practices for mathematics. These include practices within a mathematics-context that

> 1. Support work that is central to mathematics, 2. Helps to improve the learning and achievement of all students, 3. Is done frequently when teaching mathematics, 4. Applies across different approaches to teaching mathematics

> 5. Can be articulated and taught, 6. Is accessible to learners of teaching 7. Can be revisited in increasingly sophisticated and integrated acts of teaching, 8. Is able to be practiced by beginners in their field-based settings. (Ball et al., 2009, p. 461)

These criteria can provide guidance in understanding and identifying instruction within teacher education programs that will equip future elementary educators with the most effective mathematics tools and strategies to bring into their classrooms. However, we must also consider how these instructional strategies are perceived by pre-service teachers and if and how they influence a pre-service teacher's development as a future educator.

This research acknowledges not only the complexities in understanding and measuring interconnected and multidimensional constructs such as educator knowledge and self-efficacy, but it recognizes these constructs exist within various independently operating teacher preparation environments. This review of the literature constructs a theoretical framework used to examine this problem of practice, as well as provides an analysis of relevant empirical studies.

Theoretical Analysis

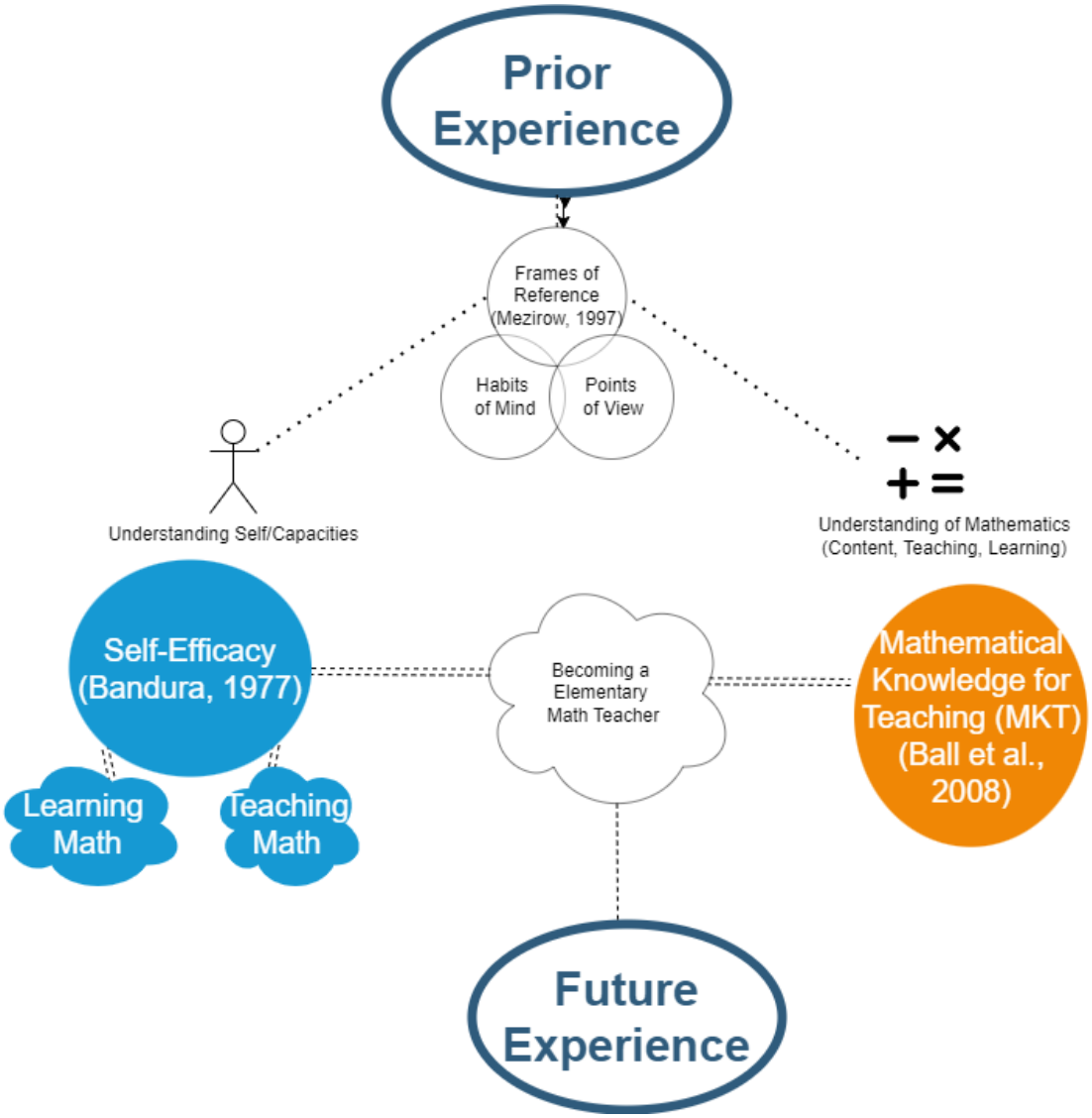
The conceptual framework used to approach this problem of practice and guide the research study integrated several existing theories. Primarily, Mezirow's Transformational Learning Theory is used to understand how emerging educators' prior experiences with mathematics influence their beliefs about themselves as learners of mathematics, as examined through Bandura's Self Efficacy Theory, as well as how their experiences influence their understanding of mathematics as a subject

area they teach, as examined through Ball et al. (2008)'s practice-based theory of Mathematical Knowledge for Teaching. Each of these theories, in addition to related theories, will be discussed in greater depth.

As indicated in Figure 2.1, experiences that individuals have had related to mathematics and learning mathematics create frames of reference (Mezirow, 1997). These frames of reference include assumptions as a result of their own mathematics understanding and assumptions of how to teach based on prior experiences as students of mathematics. These prior experiences serve as a baseline when pre-service teachers enter their teacher preparation programs. They have ideas about their capabilities to do mathematics (Bandura, 1977), as well as ideas and assumptions about the nature of mathematics learning and teaching from what they experienced as K-12 students (Cranton, 2016). The second part of Figure 2.1 considers as pre-service teachers engage in their teacher preparation program and when new in-service teachers gain additional classroom experience, their beliefs in their capabilities and knowledge of how to teach mathematics evolves and may have an impact on their future teaching experiences.

Figure 2.1

Conceptual Framework



Content Knowledge for Teaching

Teacher knowledge and knowledge subsets have been examined over the course of several decades. Shulman (1986) first suggested a model of different domains of teacher knowledge including "subject matter content knowledge," "pedagogical content knowledge," and "curricular knowledge" (p. 9). This model

differentiates components of the subsets of knowledge needed to teach. Shulman (1986) argued the dichotomy between content and pedagogy in teacher education is problematic.

Subject matter content knowledge moves beyond the facts of a subject and involves why certain pieces of knowledge are important for the field (Shulman, 1986). There is a depth of reasoning both in what and why something is important (Shulman, 1986). Curricular knowledge includes a host of resources and "alternative curriculum materials" to teach a single subject and also "lateral curriculum knowledge" which, with older students, includes knowledge of what they are learning in other courses (Shulman, 1986, p. 10). With this knowledge, a teacher can better integrate and make connections across disciplines. It also includes knowledge of content across grade ranges. A teacher has curricular knowledge when they are aware of what content precedes where their students are currently and what lies ahead of them as they continue their education in the subject area (Shulman, 1986). Pedagogical content knowledge is the "subject matter knowledge *for teaching"* (Shulman, 1986, p. 9). This includes an educator being able to present content material in a way that is effective for learning. For example, an educator should have many ways to represent material, have an understanding of knowledge of "preconceptions and misconceptions" students commonly bring with them to learning content, and have insights into the best instructional strategies for subject areas (Shulman, 1986, p. 10).

Teacher knowledge is complex, and Shulman's (1986) model provided a foundation to understand and explore the different subsets of knowledge. From a mathematics perspective, knowing mathematics is not sufficient in being a competent

mathematics educator. Subject matter knowledge is important, but teachers need to know how to use their knowledge of mathematics to teach effectively. This requires a knowledge unique to teaching that combines subject matter knowledge with knowledge of teaching. This is important to be mindful of when examining pre-service experiences. Do pre-service teachers have opportunities to develop different types of knowledge such as pedagogical content knowledge?

Mathematical Knowledge for Teaching

Building off Shulman's (1986) model for teacher knowledge and with a particular emphasis on pedagogical content knowledge, Ball et al. (2008) created a practice-based theory of mathematics content knowledge for teaching. Mathematical content knowledge further described the competencies and skills needed to teach mathematics (Ball et al., 2005; Ball et al. 2008). The researchers suggested Shulman's (1986) construct of subject content knowledge can further be divided into "common content knowledge" (Ball et al., 2008, p. 399), "horizon content knowledge" (p. 403), and "specialized content knowledge" (Ball et al., 2008, p. 400). Ball et al. (2008) also further divided and defined subsets of pedagogical content knowledge, which can be split into "knowledge of content and students," "knowledge of content and teaching" (p. 401), and "knowledge of content and curriculum" (p. 402).

Common content knowledge is the knowledge that individuals have outside of teaching mathematics. Every day adults are expected to have basic mathematics skills and knowledge of how to calculate proficiently; this is considered common content knowledge. Specialized content differs in that it goes beyond the basic understanding of mathematics that common adults possess. Specialized content knowledge is unique

to teaching and requires strategic use of mathematical knowledge that individuals outside of teaching do not need. Specialized content knowledge includes breaking down mathematics concepts and making them accessible to students (Ball et al., 2008). This may include thoughtful decision-making about how to most appropriately introduce and explain concepts and key mathematical ideas (Ball et al., 2008). Furthermore, this knowledge may include an explanation of why and how a procedure works and the underlying concepts and connections to other concepts and ideas. Teachers' specialized content knowledge enables the careful selection of examples and instructional activities, that promote effective mathematics learning (Ball et al. 2008; Ball et al. 2005). The third aspect of subject content knowledge is horizon content knowledge, which encompasses a big-picture understanding of mathematics content across grade levels (Ball et al., 2008). Horizon content knowledge enables a teacher to build off prior knowledge and set a foundation for later concepts and mathematical relationships (Ball et al., 2008). A mathematics educator must possess and utilize each of these subsets of subject matter knowledge, yet there are additional knowledge domains proficient educators use as well.

Pedagogical content knowledge is the second major aspect of mathematical knowledge for teaching and consists of three sub-domains: knowledge of content and curriculum, knowledge of content and teaching, and knowledge of content and students. Ball et al. (2008) maintained Shulman's (1986) construct of curricular knowledge within the framework, as a standalone category, knowledge of content and curriculum, yet note further work is needed to determine the appropriateness of this distinction or whether it may fall within the boundaries of knowledge of content and

teaching (Ball et al., 2008). Ball et al. (2008) described knowledge of content and teaching as the integration of teaching and mathematics knowledge. This involves the knowledge of planning a sequence of lessons and scaffolding mathematics tasks. Knowledge of content and teaching also involves instructional decision-making, such as on-the-spot choices to advance the depth of learning (Ball et al., 2008). Lastly, knowledge of content and students combines knowledge of students and knowledge of mathematics, such as interpreting student responses and responding appropriately (Ball et al., 2008; Ball et al., 2005). An important aspect of this domain is recognizing student misconceptions and errors and being equipped with knowledge of how to respond to these errors and misconceptions (Ball et al., 2008).

Teacher knowledge is complex and involves multiple layers and dimensions. Ball and colleagues (2008) acknowledged the challenges in defining what constitutes a particular domain(s) of knowledge and noted the lines may blur between subsets, yet this theoretical model has provided researchers and scholars a baseline for further research into teacher knowledge necessary for teaching mathematics. This model can be used to examine pre-service teacher education programs, in particular, insights into the types of knowledge that are included and emphasized within coursework and programs. Decades ago, Shulman (1986) brought awareness to the divide between general pedagogy and subject matter content, and yet this divide is still present and evident in teacher education programs today (Canadas, 2013; Strawhecker, 2005). In addition to developing mathematical knowledge for teaching, it is important for emerging elementary teachers to also have beliefs in their own mathematics abilities

and develop positive beliefs regarding their capacities to teach mathematics to their future students.

Self-Efficacy Theory

Bandura (1994) defined self-efficacy as "people's beliefs about their capabilities to produce designated levels of performance that exercise influence over events that affect their lives" (p. 72). Self-efficacy can impact individuals in a variety of ways, such as influencing internal feelings and thoughts, including "cognitive processes," and it can also influence an individual's choices, behavior, and actions (Bandura, 1994, p. 72). Those individuals with strong self-efficacy are more inclined to engage in challenging activities and persist, whereas those individuals with weak self-efficacy are more inclined to avoid challenging activities and give up more readily (Bandura, 1994). This avoidance behavior then becomes problematic for individuals.

When individuals avoid perceived negative situations or experiences, they do not develop "coping skills," yet when individuals take part in these stressful environments, they develop the ability to cope (Bandura, 1977, p. 194). Efficacy expectations are a major contributing factor into whether someone engages in an activity, how much effort they put forward, and how long they persist in the activity (Bandura, 1977). Applied to experiences with mathematics, a person with strong mathematics self-efficacy is at a higher likelihood to embrace mathematics, even if the task appears challenging. This is in contrast to an individual with weak self-efficacy, who may avoid mathematics tasks and fail to persist under challenging circumstances.

Bandura (1977) identified four different factors that can influence self-efficacy, "performance accomplishments," "vicarious experiences," "verbal

persuasion," and "emotional arousal" (p. 195). Performance experiences have an influence because as an individual succeeds in tasks, their expectations for success increase. In contrast, as an individual persistently fails, their expectations decrease (Bandura, 1977). Simply put, successful experiences cause individuals to believe they are capable, leading them to persist in future tasks. If individuals face challenges, especially prematurely in self-efficacy development, this influences their self-views of their capabilities to perform tasks, leading them to not develop resiliency to engage in challenging tasks (Bandura, 1977). Strong efficacy expectations allow someone to fail from time to time and not become discouraged, so the development of self-efficacy through mastering content is important and the use of scaffolded learning activities may be useful to build self-efficacy (Bandura, 1977).

Vicarious experiences are another mechanism to develop self-efficacy and involve being able to see someone else successfully achieve in an environment or task without consequence (Bandura, 1977). These experiences can influence efficacy expectations and show individuals they can also be successful. Furthermore, it is more beneficial to see someone modeling overcoming their own difficulties than hearing from those who have not struggled (Bandura, 1977). A similar indirect mechanism to develop self-efficacy is verbal persuasion, which encompasses being told it is possible to overcome something (Bandura, 1977). However, self-efficacy expectations are not as strongly developed through verbal persuasion as through actual personal experiences (Bandura, 1977). There is no guarantee an individual will believe what they are being told about their abilities if their own experiences demonstrate repeated

failures and poor experiences. This re-emphasizes the important role experiences have on self-efficacy development.

Finally, Bandura (1977) described emotional/psychological arousal as an influential factor of self-efficacy. Psychological arousal determines whether a person will be stressed or anxious about a given task or phenomenon. This means that simply thinking about an anxiety or fear-provoking activity can create a higher level of arousal than the level of anxiety of the event itself (Bandura, 1977). Teaching coping skills for such an event is important because avoiding an activity does not allow for the development of coping mechanisms. If an individual believes they are better off than they once thought, they are less likely to be in increased arousal.

Cognitive processes are complex and multi-dimensional and if an individual's lived experience is counter to efficacy expectation, this may not cause a behavior change (Bandura, 1994). Individual perceptions are an integral component of self-efficacy and its effects. A person's self-efficacy will increase if their performance is viewed as related to their abilities. Accordingly, if a person believes their failures are linked to their abilities, this can decrease self-efficacy (Bandura, 1977). Another important notion within Bandura's framework is that efficacy expectations are not static and need to be measured over time (Bandura, 1977). Experiences that allow an individual to overcome challenging tasks or make progress after facing challenging circumstances can be potent building blocks in self-efficacy development.

Another aspect of self-efficacy to consider is the broader implication self-efficacy has on an individual's educational and career trajectory. For example, Bandura (1994) indicated a student's choice of career path is influenced by their self-

efficacy. Once engaged in a college or job training program, self-efficacy influences the extent to which a future employee gains "the basic cognitive, self-management and interpersonal skills on which occupational careers are founded" (Bandura, 1994, p. 13). This clearly has implications for pre-service educators in the ways they provide the aforementioned self-efficacy development factors within their classroom environments. Teacher self-efficacy also has impacts on students' performance and drive to learn (Bandura, 1994). Bandura (1994) further argued environments that reinforce competition and rank are damaging to self-efficacy, and environments that promote individual growth and cooperative learning build self-efficacy. Furthermore, because student self-efficacy is influenced by educators' efficacy, it can impact students' academic achievement and future career trajectories. These are all important considerations pertinent to teaching.

Lastly within teacher preparation programs, Bandura's theory has implications for teacher educators to consider the self-efficacy of pre-service educators. Specifically, it is noteworthy to consider the mathematics self-efficacy of future educators: Is it strongly developed, or do future educators have tendencies to avoid mathematics? If pre-service educators have strong beliefs in their own mathematical abilities and the abilities of their students, this may influence their ability to teach and value mathematics. There may be opportunities within teacher preparation programs to unpack the mathematics self-efficacy of pre-service teachers and promote opportunities to strengthen the mathematics self-efficacy of these future educators. One lens through which to consider pre-service educators' prior experiences and self-efficacy is adult learning theory.

Adult Learning Theory

Although the term, “andragogy” has roots dating back to the 1800s, it became increasingly popularized and made relevant through the development of Knowles’ adult learning theory (Knowles, 1984). Knowles (1984) created a distinction between children’s and adults’ learning processes and indicated adults have different learning requirements than those of children. Andragogy is comprised of the following key underlying assumptions: “changes in self-concept,” “the role of experience,” “readiness to learn,” and “orientation to learning” (Knowles, 1984, pp. 45-47). Knowles and colleagues later identified “motivation” and “need to know” as additional components of the theory (Knowles et al., 2005). An understanding of how adults best obtain and make meaning of knowledge is an important consideration for teacher educators charged with preparing pre-service educators for classroom teaching.

As a person grows and proceeds through life, their self-concept evolves. Self-concept changes from dependency on others to more self-directed (Knowles et al., 1984) Knowles indicated that as adults enter the workforce or engage in professional-focused programs beyond K-12, they envision themselves as self-directed learners. These self-directed adults respond positively to being viewed and treated in this way and respond negatively when they perceive they are viewed or treated similarly to dependent children (Knowles, 1984). This notion is relevant to teacher education programs, which are meant to prepare teacher candidates for a specific profession. According to Knowles’ theory, self-directed learning activities are warranted and most effective.

Experiences that adults bring to the educational environment and learning process are also important considerations within adult learning theory. Adults have a wealth of experience, which can serve as a resource for others in the classroom. Additionally, the experiences of each adult individual are the foundation from which the learner engages with new content. Adults become defined by their lived experiences, as these serve as a source of truth (Knowles, 1984). In effect, adults feel valued if their experiences are utilized and valued within the classroom. Consequently, they feel personally undervalued when their experiences are not heard or acknowledged (Knowles, 1984). Adult experiences cause influential differences in adult cognitive approaches to problem-solving and thinking (Knowles, 1984). These differences are arguably more important to be aware of in adults compared to children because as adults age, they have had more experiences that impact the way they interpret the world (Knowles, 1984). Knowles (1984) indicated the ways in which adults engage with new content matter. Experiential learning should be prioritized over more traditional methods (Knowles, 1984). Adult learners utilize their experiences and incorporate this into their learning when teachers offer "discussion, laboratory, simulation, field experience, team projects, and other action learning techniques" (Knowles, 1984, p. 46) to stimulate and promote effective adult learning.

It is not only important to recognize and incorporate adults' experiences into the learning environment, but it is also most effective to embed prior knowledge within the design of the learners' experiences through the use of strategic instructional experiences, such as field experiences (Knowles, 1984). In thinking specifically of pre-service educators who will be teaching mathematics, it becomes relevant to

consider future educators' past experiences as mathematics learners and the most fruitful experiences to actively build upon this knowledge within the teacher education program.

As an individual moves from child to adult, there is a shift in their readiness to learn new information. Adults are ready to learn information relevant to their current developmental stage, such as a new or anticipated professional role (Knowles, 1984). The timing of learning experiences is important and needs to be in alignment with where the learner is developmentally. This alignment occurs when the learner observes the relevancy of the content to their unique needs (Knowles, 1984). Knowles (1984) indicated that an adult learner educator can create opportunities to facilitate this observance of relevancy within the learning environment rather than waiting for all adult learners to become self-aware of applicable relevancy. Relatedly, adult learning theory assumes adults need to have clear reasons why they are learning material (Knowles, et al., 2005). These assumptions become important for teacher educators to consider and utilize in their organization of teacher education program curriculum and throughout instruction within pre-service educator courses.

Another assumption of adult learning theory is an adult's orientation to learn. Knowles distinguished adult orientations to learn from children's by describing that a child's orientation is driven by the subjects children learn. The material learned sets the foundation for further knowledge which allows the child to achieve the next educational milestone (Knowles, 1984). This diverges from an adult's orientation to learn, in that adults are oriented around a problem and have a desire to actively apply their knowledge. Lastly, consideration of adults' motivation to learn is identified as an

assumption of Knowles' theory (Knowles et al., 2005). As traditional pre-service educators transition to adulthood or non-traditional career-changers focus on a new career path, they become more highly focused on their professional program and career. With this shift, attention to changes in intrinsic motivation and orientation to learn become relevant considerations for teacher educators.

There have been several criticisms of adult learning theory. Some have argued that andragogy should not be classified as a theory at all, claiming it is a set of guidelines or best practices rather than a robust theory (Merriam, 2001). Other critiques are centered on the underlying assumptions of andragogy (Merriam, 2001). Scholars have argued that many of these assumptions may not be unique to adults only because children's learning may also align with these assumptions, including possessing motivation, constructing knowledge from their own experiences, and thriving in a self-directed environment (Merriam, 2001). Conversely, adults may not always meet the underlying assumptions of andragogy. Some have argued adults may in fact require more teacher-directed learning in certain instances, lack motivation at times, and have limited experiences or experiences that are counterproductive to learning (Merriam, 2001).

Another criticism is that adult learning theory considers adult learning at an individual level without the broader consideration of the individual in an environment with social, cultural, and other interconnected factors (McGrath, 2009). Alfred (2002) argued that an individual-focused view of a learner "presents a static view of learning and fails to account for how issues of positionality influence the learner's experience" (p. 4). It is necessary to consider how one's generation, race, ethnicity, gender, sexual

orientation, and socioeconomic status, in addition to other social and cultural differences, shape and influence one's experiences (Alfred, 2002).

These criticisms warrant attention, particularly the various influential factors that impact individuals differently. However, there are tenets of adult learning theory that are meaningful considerations for pre-service teachers' education. Considering that pre-service educators' lived experiences can prove beneficial to their growth, past mathematics experiences can be viewed as the foundation for pre-service educators' mathematics learning. This must be taken into consideration and utilized to enhance pre-service educators' mathematical knowledge for teaching. Considering Knowles' theory in relation to self-efficacy theory, there is potential for pre-service educators to enter pre-service programs with strong or weak mathematics self-efficacy. Their past experiences have influenced their perceptions of their mathematical abilities and these are crucial to recognize. In terms of adult learning theory, these past mathematical experiences should be acknowledged, validated, and used as a starting point for learning. Therefore, providing opportunities for future educators to examine their prior mathematics learning and build self-efficacy are worthy of attention in teacher education programs. Another adult learning theory, transformational learning theory, considers ways in which adult learners can engage in learning environments that enable them to reflect on their experiences and beliefs and evolve their thinking.

Transformative Learning Theory

Mezirow (1997) introduced an alternative theory of adult education, transformative learning theory. Similar to Knowles' theory, a tenant of transformative learning theory is the importance of lived experiences. Adults' experiences create what Mezirow (1997) called "frames of reference" or "structures of assumptions through which we understand our experiences" (p. 5). An adult's default response is to agree with ideas that support their own experiences and dismiss ideas that are not congruent with their experiences (Mezirow, 1997). As the name suggests, there is an opportunity to transform or change these viewpoints. Examining frames of references further, Mezirow (1997) differentiated two domains: "habits of mind" and "point of view" (p. 5). Habits of mind are influenced by an individual's culture and environment and become more deeply rooted and ingrained within an individual over time. A point of view is the result of one's habit of mind and includes "feelings, beliefs, judgments, and attitudes" (p. 6). Points of view have the potential to change more readily based on one's experiences. Mezirow (1997) argued that when individuals reflect on their assumptions, there is room to change and grow.

When adult learners are involved in communicative learning, they are engaged in the critical reflection of their beliefs, thoughts, and feelings (Mezirow, 1997). Additionally, adults can engage in discourse that can unpack a point of view by examining different arguments (Mezirow, 1997). The important notion with transformative learning is that adults have the ability to change and grow by reflecting on their assumptions. Educators of adults can create opportunities within their

classrooms for reflection and discourse. The teacher educator can also model reflective practices for adult learners.

Taylor (2008) described multiple interpretations and evolutions of Merizow's transformative learning theory. As is observed with criticisms of Knowles' adult learning theory, a primary concern is that the theory centers on the individual without regard to the influence of social and cultural differences (Taylor, 2008). However, multiple scholars have interpreted and evolved the theory in other ways to bring awareness to these differences. Taylor (2008) stated,

> those views that recognize difference (social emancipatory, culturally relevant narrative, race-centric, and planetary) place much greater emphasis on positionality (where one's 'position' is relative to race, class, gender, sexual orientation) and its relationship to both the process and the practice of transformative learning. (p. 10)

Another criticism is that a transformative learning experience may be impeded if the adult educator fails to acknowledge individual learning preferences and comfort levels. For example, a course could place too great an emphasis on reflection by means of journal writing, when this may not be a preferred learning style of all students. These critiques do not negate the value of facilitating a transformative learning process, yet they illustrate the complexity of enacting such changes and the depth of and interconnectedness of one's experiences.

Transformative learning theory posits the need to provide pre-service educators with opportunities for critical reflection within their teacher education programs (Mezirow, 1997). This may be necessary for multiple reasons. Pre-service educators

have developed their frames of reference around mathematics learning and these frames of reference include assumptions of their own mathematics understanding and assumptions of how to teach mathematics based on prior experiences as students. In considering the latter assumption, pre-service educators may need to critically reflect on their own mathematics learning when provided new insights into best practices and recommendations for teaching mathematics. The other important aspect of Mezirow's theory is the compounded nature of an adult's experience. To consider a pre-service educator who comes to a teacher education program with a minimum of 12 years of educational experience as a student of mathematics, it is realistic to expect deeply rooted habits of mind, and in effect points of view, in relation to mathematics. This point merits mention for the mere fact that intentional efforts may be necessary to change attitudes, beliefs, and feelings about and towards mathematics. A particular area of consideration is pre-service teachers' own beliefs about the nature of learning and teaching mathematics and whether this is aligned with constructivist teaching.

Constructivism in Math Education. Although different interpretations and views on constructivism exist, constructivist instructional strategies are central to recommended mathematics best practices (Clements & Battista, 1990; NCTM, 2014). A key idea within constructivism is that individuals construct learning from their experiences and their knowledge builds on prior experiences (Clements & Battista, 1990; Noddings, 1980; Piaget, 1970). Constructivist teaching uses this idea to relate new learning to existing knowledge. A teacher in a constructivist classroom is a facilitator of learning and is aware of and values students' prior knowledge (Noddings, 1980). The constructivist educator strategically scaffolds learning opportunities for

students, provides opportunities for collaboration, and values a student-centered learning environment (Noddings, 1980). Clements and Battista (1990) highlighted the differences between these constructivist values from those of the traditional mathematics educator, who relies on premises of "transmission" and "absorption views of teaching and learning" (p. 6).

Constructivism in mathematics education allows students to expand their mathematical thinking through a process and understand how concepts and principles build upon one another (Clements & Battista, 1990). Additionally, constructivism in mathematics aims for students to become empowered by their learning, rather than expecting their teacher to transmit knowledge to them (Clements & Battista, 1990). One could argue these are worthy principles and instructional strategies to support competent mathematics learners and problem solvers, yet there remains the concern of what educators must do to bring constructivism into the mathematics classroom. According to Clements and Battista (1990),

> Constructivist teachers must be able to pose tasks that bring about appropriate conceptual reorganizations in students. This approach requires knowledge of both the normal developmental sequence in which students learn specific mathematical ideas and the current individual structures of students in the class. Such teachers must also be skilled in structuring the intellectual and social climate of the classroom so that students discuss, reflect on, and make sense of these tasks. (p. 35)

It is evident that to successfully enact constructivism within the mathematics classroom, teachers need to bring a depth of mathematical knowledge for teaching.

Clements and Battista (1990) alluded to the multiple dimensions of content knowledge an educator must have to engage in meaningful mathematics learning, such as specialized content knowledge and mathematical pedagogical content knowledge.

Glaserfeld (2001), who is known for his radical constructivism approach, proposed several suggestions teachers can adopt to bring constructivism to their classrooms. An important component in this is that the teachers' role is to create an environment that does not reinforce the notion of a right and wrong answer, correct or incorrect approach, but instead creates a place in which students' thinking drives learning (Glaserfeld, 2001). Students' ideas are not only important but are the foundation on which the instruction is built. Glaserfeld (2001) also illustrated the importance of educators' mathematical knowledge by stating, "It is not sufficient for teachers to be familiar with the subject matter of the curriculum; they also have to have a repertoire of didactic situations in which the concepts that are to be built up can be involved" (p.10). Glaserfeld (2001) echoed Ball and colleagues (2008) in the need to develop specialized content knowledge and mathematical pedagogical content knowledge. Furthermore, tenets of self-efficacy development are evident within Glaserfeld's (2001) call to educators. Particularly, Glaserfeld (2001) emphasized the importance of an educator's response to an incorrect solution. He stated, "disregarding it, is a sure way to demolish whatever spark of motivation they had. And then it is not surprising that their willingness to tackle new tasks disappears" (p. 11).

Not only do pre-service educators need to become equipped with constructivist learning techniques to use in their future classrooms, but it is worth considering pre-service educators' own past learning experiences. Lastly, it may be necessary to

explore the instructional strategies of teacher educators: Are teacher educators modeling student-centric learning, collaborative discourse, facilitation of learning, and other cornerstone elements of constructivist teaching within the pre-service classroom?

In examining these theories together, there are elements that link them with one another. Yet, given the multiple considerations for important components of teaching mathematics, developing self-efficacy, and effectively educating adult learners, teacher educators have a tall order to fill in a few short years. It may be in the best interest of teacher educators and teacher preparation programs to consider core practices that are going to promote productive change and prepare pre-service teachers to facilitate mathematics learning for their students. However, the ways in which pre-service teachers internalize these experiences remains readily unavailable in the literature. There is a great need to more clearly understand how emerging elementary teachers perceive various practices and what types of experiences they attribute to enhanced self-efficacy and growth in their mathematics knowledge.

Empirical Analysis

Mathematical Knowledge for Teaching

Educator knowledge has been of great interest over the past decades, and in particular in mathematics, as the United States has undergone efforts to improve mathematics achievement in students (National Council of Teachers of Mathematics [NCTM], 2014). Shulman's (1986) constructs, specifically, pedagogical content knowledge, and subsequently Ball et al.'s (2008) mathematical knowledge for teaching framework precipitated a series of investigations with the intent of better

understanding the knowledge required of educators to teach mathematics effectively. Mathematics content knowledge for teaching has been found to be a significant predictor of student mathematics achievement, with the teachers with the greatest mathematical knowledge to teach mathematics being linked to students with the greatest mathematics achievement (Hill et al., 2005). Hill et al. (2005) found that mathematical knowledge for teaching was such a powerful predictor of achievement that it rivaled the effects of socioeconomic status on student achievement, suggesting mathematical knowledge for teaching is influential on student outcomes. These findings underscore the importance of developing mathematical knowledge for teaching during teacher education programs to prepare future educators with a depth of mathematical knowledge that will influence their future students.

Pre-service teachers need a solid understanding of not only procedures but of the conceptual underpinnings and relationships within mathematics. Leading organizations in mathematics education have emphasized the importance of students developing both procedural knowledge of how to carry out mathematical operations, and knowledge of underlying mathematical concepts (NCTM, 2014). This means pre-service educators must be proficient in procedures and fundamental concepts. NCTM (2014) emphasizes that procedures are not isolated from mathematical concepts, but rather "when procedures are connected with underlying concepts, students have better retention of procedures and are more able to apply them in new situations" (p. 42). Although both procedural and conceptual knowledge are important to develop, there remains a delicate balancing act, and in some instances disagreement over

emphasizing initial concepts only or integration of concepts and procedures within a lesson (Rittle-Johnson et al., 2016).

Rittle-Johnson et al. (2016) examined student outcome differences between instruction that focused on solely concepts compared to instruction that integrated concepts and procedures within a single lesson. Findings suggested that students who received conceptual-only instruction showed greater development of conceptual and procedural knowledge and retained this knowledge over time (Rittle-Johnson et al., 2016). Not only do pre-service teachers need to have procedural and conceptual knowledge, but they must be equipped with how best to integrate this knowledge effectively into their instruction.

Fuentes et al. (2014) examined confidence in pre-service educators' understanding of mathematics and science concepts and procedures, as well as confidence in teaching these. Findings indicated means were significantly higher for answering procedural questions compared to teaching those concepts, indicating more confidence in teaching procedures than concepts (Fuentes et al., 2014). Pre-service educators expressed frustration in having to teach in different ways than they learned, for example "borrowing" (Fuentes et al., 2014), which touches upon the theoretical idea that pre-service educators may need to re-learn mathematics ideas deeply rooted in their own educational experiences and frames of reference (Mezirow, 1997). A limitation of this mixed methods study is the sample size was small and further research is needed to not only inform pre-service educators of the importance of conceptual and procedural understanding but on ways to develop these within teacher education programs and discuss how future educators can incorporate both concepts

and procedures in their own instruction (Fuentes et al., 2014). Careful attention must be placed on experiences within teacher education programs that provide opportunities for critical reflection of past learning experiences and developing an active understanding of practices and concepts that build depth of understanding in mathematics.

In an international quantitative study, Van Steenbrugge et al. (2014) explored pre-service educators' procedural and conceptual knowledge of fractions and found procedural knowledge of fractions was greater than conceptual knowledge. Furthermore, the gaps in educators' conceptual fraction knowledge were the same gaps observed in students' conceptual fraction knowledge (Van Steenbrugge et al., 2014). Although perhaps not surprising, this finding is alarming, suggesting limitations in educator knowledge are passed on to students, creating a cyclical pattern of limited mathematical knowledge. Even provided commonalities in limited fraction knowledge among pre-service educators abroad and in the states (Newton, 2008), the contextual nature of the Van Steenbrugge et al. (2014) study needs to be taken into consideration to generalize its findings to pre-service educators in the United States. Additionally, provided the quantitative nature of this study, it is difficult to understand the depth of pre-service teacher misconceptions, which could provide insight into how to better prepare future educators during their teacher education programs.

In a longitudinal quantitative study, Swars et al. (2007) found a discrepancy between pre-service educator mathematical teaching efficacy and mathematical specialized content knowledge, essentially indicating that pre-service educators were largely unaware of what they did not know. Norton (2017) examined pre-service

elementary educator confidence in mathematics content knowledge and confidence to teach mathematics concepts and observed pre-service educators were also unaware of their inability to complete mathematics problems with accuracy. Norton (2017) further highlighted the concern that these future educators were near the end of their teacher education program, having received high academic marks, indicating pre-service teachers may not be sufficiently developing mathematics knowledge during their teacher education programs. A limitation of these studies is the quantitative methods and Likert-based surveys used to examine pre-service educators' understanding and confidence. Qualitative data could provide more insight into pre-service educators' perceptions of themselves as mathematics educators and awareness of their own level of mathematical knowledge for teaching.

The types of mathematical knowledge covered in pre-service education have been examined to an extent in the literature. Koponen et al. (2017) examined the mathematical knowledge for teaching components within a secondary-level mathematics teacher education program in Finland. They found that there was equitable representation between subject matter knowledge and pedagogical knowledge, but within the subsets of these two branches of mathematical knowledge, there were variations in emphasis (Kopenen et al., 2017). Specifically, in terms of subject matter knowledge, common content knowledge was emphasized to a greater extent than specialized content knowledge, implying that future educators may not be exposed to the most useful types of mathematical knowledge for teaching (Kopenen et al., 2017). In terms of pedagogical content knowledge, knowledge of content and teaching was represented more than knowledge of content and students (Kopenen et

al., 2017), which implies there was limited focus on how students learn mathematics. Kopenen et al. (2017) observed content regarding identifying struggling students but found limited content on understanding why students may be struggling and identifying misconceptions. Further research is needed to observe the presence of subject matter knowledge and pedagogical content knowledge and their subsets in other teacher education programs, specifically elementary teacher education programs in the United States, and how opportunities to experience these subsets of knowledge are perceived by pre-service teachers.

The development of mathematical content knowledge and mathematical pedagogical content knowledge has also been examined to an extent in the literature. Several studies have examined the use of and influence of different instructional strategies on these knowledge variables (Carney et al., 2016; Depaepe et. al, 2018; Krawec et al., 2014; Polly et al., 2013). Polly et al. (2013) explored how a professional development program focused on instructional pedagogies influenced in-service elementary educators' content knowledge, beliefs, and instruction. The specific instructional strategies and intentions of the professional development were to "deepen teachers' capacity to teach through cognitively demanding tasks and an inquiry-based approach" (Polly et al., 2013, p. 3). Findings indicated mathematical content knowledge increased significantly and beliefs about mathematics instruction shifted from a transmission viewpoint to a more discovery and constructivist viewpoint. Polly et al. (2013) did not observe immediate changes in how educators enacted their instruction and the authors argued this takes time to develop.

Carney et al. (2016) explored the influence a professional development course focused on instructional strategies had on mathematical knowledge for teaching, self-efficacy, and beliefs. The course took a constructivist approach to instruction and classroom practices, including participant-led activities and discussions, and focused on five key instructional practices modeled by the instructor: "taking students' ideas seriously, pressing students conceptually, encouraging multiple strategies and models, addressing misconceptions, and focusing on the structure of mathematics" (Carney et al., 2016, p. 542). The findings suggested significant increases in knowledge, efficacy, and beliefs, with the most pronounced mathematical content knowledge growth among K-3 educators. Carney et al. (2016) acknowledged limitations in quantitative data for understanding knowledge and efficacy changes and stated, "by operationalizing these constructs as quantifiable variables, we lose important information about the multidimensional and interactive nature of these constructs at the individual level" (p. 560). The Carney et al. (2016) study provides insights into key instructional strategies that appear to have an influence on mathematical knowledge for teaching and self-efficacy and these constructs within the context of the instructional interventions could be explored further at an individual level through including qualitative data.

Krawec et al. (2014) engaged in a study examining professional development, which focused on problem-solving strategies, and its impact on the educator participants' students. Findings of this study suggested explicit instruction during professional development benefited the teachers, especially in developing a paradigm shift from a procedural-focused viewpoint to a place in which educators were more critically focused on problem-solving and their own thinking processes (Krawec et al.,

2014). Another important finding was that the teachers who received training in seven cognitive processes and three metacognitive strategies had students in their classes that demonstrated significant growth in the area of problem-solving (Krawec et al., 2014). Not only were benefits observed in educator growth, but there were observable impacts on their students. Studies on educator professional development highlight efforts to modify mathematical pedagogical approaches and build depth of knowledge in mathematics (Carney et al., 2016; Krawec et al., 2014; Polly et al., 2013). Although the instructional practices are informative, their participants must be considered in contrast to pre-service educators. Veteran in-service educators have experience teaching and working with learners of mathematics. The level of experience may impact educators' development of mathematical knowledge for teaching in different ways.

Ire et al. (2018) suggested mindsets may change depending on circumstances, and teacher education programs create an environment that promotes a growth mindset and professional growth. Perhaps, in-service professional development programs do not replicate the same growth-orientated environment or perhaps in-service educators place greater importance on their in-classroom experiences than on suggestions and new initiatives presented by professional development instructors. Krawec et al. (2014) indicated a disconnect between research-based practices and providing guidance to teachers on how to best implement them. Given all educators are asked to do, particularly elementary educators who are generalists and expected to be competent in multiple subject areas, it may become challenging to enact meaningful and large-scale changes to mathematics instruction provided multiple demands. As

Polly et al. (2013) indicated in the delayed observance of in-service educator instructional changes, it is reasonable to assume these changes take time and require ongoing support. Pre-service educators may not only be naturally within a productive learning-centric environment but also within an organized program that has the potential to promote ongoing support and spend more time on research-supported instructional strategies and high-leverage practices.

Strawhecker (2005) examined varying types of teacher preparation and the influence those have on pre-service educator mathematical content knowledge. Findings suggested field experiences during mathematics preparation training impacted teachers' knowledge and ability to understand students' mathematical thinking. Interestingly, Kulgemeyer & Reise (2017) found field experiences had little impact on the academic performance of pre-service educators. There was variation among student experiences during internships dependent on cooperating teachers' pedagogical and instructional views. Forzani (2014) also argued field experiences vary in the extent to which they engage future educators in different mathematics concepts and practices. Forzani indicated, "novices might spend months in student teaching or participating in a residency program and never learn how to lead a productive whole-group discussion" (p. 358). Reid and Reid (2017) explored pre-service educator mathematical knowledge for teaching and its development through a mathematics course. Results indicated mathematical knowledge for teaching improved through coursework. In particular, pre-service educators reported valuing opportunities to reflect on their initial performance scores to set goals for themselves and see growth

over the length of the course, and evidence suggested beliefs and attitudes towards mathematics changed (Reid & Reid, 2017).

Depaepe et al. (2018) utilized a framework of conceptual change to explore the use of specific instructional strategies on pre-service educators' content knowledge and pedagogical content knowledge. The intervention group, which focused on conceptual change and employed specific instructional strategies, including vignettes and concept cartoons, was compared to a control group, which did not use these instructional strategies (Depaepe et al., 2018). The control group made little progress on content knowledge and pedagogical content knowledge, whereas the intervention group showed small improvements in content knowledge and pedagogical content knowledge. The findings suggested being confronted with student misconceptions through a conceptual change framework may have led pre-service educators to see their own misconceptions of mathematics concepts (Depaepe et al., 2018). The growth in content knowledge was greater than the change in pedagogical content knowledge. It appears the instructional strategies had a positive effect compared to the control group; however, there was greater growth observed in content knowledge compared to pedagogical content knowledge (Depaepe et al., 2018). Although these studies provide insights into instructional strategies that enhance educators' mathematical content knowledge (Carney et al., 2016; Depaepe et. al, 2018; Krawec et al., 2014; Polly et al., 2013), researchers argue the need to explore which specific elements within teacher education and professional development influence changes in knowledge and instructional beliefs (Polly et al., 2013).

Past research has suggested content knowledge development as a precursor to pedagogical content knowledge (Koponen et al., 2017). Ball et al. (2008) described content knowledge as "foundational to teacher competency" (p. 404) and Koponen et al. (2017) suggested the need to develop an understanding of the subject content before using the knowledge of the subject to teach effectively. Depaepe et al. (2018) pointed to a need to separate and individually examine pedagogical content knowledge development and content knowledge development and noted limitations in their study. Specifically, the research design did not allow for a detailed understanding of direct experiences that attributed to knowledge gains nor did it consider other contributing factors (Depaepe et al., 2018).

Kulgemeyer and Riese (2017) explored the influence of pedagogical content knowledge, content knowledge, and beliefs about the academic performance of pre-service educators. Results indicated that both content knowledge and pedagogical content knowledge were important to academic performance, but pedagogical content knowledge was a mediating factor that allowed for the transfer of content knowledge to students (Kulgemeyer & Riese, 2017). Murray et al. (2018) argued the need to question the assumed connection between content knowledge and pedagogical knowledge and the accepted notion that content knowledge leads to the development of pedagogical content knowledge. Their study found contradictory findings to the assumption that content knowledge and pedagogical content knowledge are closely related (Murray et al., 2018). Murray et al. (2018) observed pre-service educators with low content knowledge but high pedagogical content knowledge, indicating content knowledge as a mechanism for pedagogical content knowledge is a flawed

assumption. Furthermore, Murray et al. (2018) suggested the number of courses may not in fact be an accurate predictor of content knowledge and pedagogical content knowledge because of the weak relationship found between these knowledge domains and opportunities to learn. Murray et al. (2018) and Kulgemeyer and Riese (2017) discussed the need for further research to examine the intricacies of mathematical knowledge for teaching, the interrelationships between content knowledge and pedagogical content knowledge, and the instructional practices within courses that influence knowledge development. Further research examining the development of these constructs from the perspectives of pre-service teachers will enable teacher education programs to establish robust curriculum and effective instruction for pre-service educators. Fauskanger (2015) additionally criticized prior mathematical content knowledge research findings and questioned the effectiveness of survey instruments that measure mathematical knowledge for teaching. Fauskanger (2015) found differences in mathematical content knowledge for teaching multiple-choice survey results and constructed responses, indicating multiple-choice instruments may not be the most accurate and appropriate tool to measure teacher knowledge.

Overall, studies have demonstrated that mathematical knowledge for teaching is complex, yet it is an important component for emerging elementary educators to develop given the impacts on student outcomes. Some studies explored within their research the influence instructional strategies have on future educators during pre-service programs (Althauser, 2018; Depaepe, 2018) or in some cases on in-service teachers during professional development experiences (Polly et al., 2014; Carney et al., 2016), yet this is an area that needs to be developed further. Pulling from these

empirical sources and theoretical perspectives focused on teacher education experiences and applicable, influential instructional strategies, can be useful for understanding how pre-service teachers develop specialized content knowledge, and ways to blend theory and practice in mathematics teaching. However, little is known about how these experiences are understood from the perspectives of the pre-service teachers and how they understand their own development of mathematical knowledge during their teacher preparation programs. Additionally, more research is needed to understand how the prior experiences of pre-service teachers influence how pre-service teachers engage in learning mathematics for teaching. It may be necessary to deconstruct elements of traditional mathematics from pre-service educators' own experiences. With adult learning theories in mind, it is possible that traditional transmission views and teacher-directed instruction are deeply embedded constructs within pre-service educators' frames of reference. And if pre-service educators are entering teacher preparation programs with limited mathematical content knowledge, there is a need to address these concerns within the curriculum and through intervention techniques before the end of the program.

Beliefs, Self-Efficacy, Confidence

Mathematics Anxiety. It is not uncommon to hear individuals speak negatively about mathematics as a subject or their own perceived poor mathematics skills and abilities. For some, these negative feelings toward mathematics create anxiety. Bekdemir (2010) defined mathematics anxiety as "an illogical feeling of panic, embarrassment, flurry, avoidance, failing and fear, which are physically visible, and which prevent solution, learning, and success about mathematics" (p. 312). Hopko

et al. (2002) stated that "math anxiety is characterized by feelings of apprehension and tension concerning manipulation of numbers and completion of mathematical problems in various contexts" (p. 648). Mathematics anxiety can cause both physical and emotional responses (Finlayson, 2014). Root causes of mathematics anxiety in pre-service teachers include "instructors' hostile behavior, exam anxiety, inadequacy of instructors, peer pressure, negative attitude toward mathematics, insufficiency, students' personality type, difficulty of contents, school and surrounding context" (Bekdemir, 2010, p. 324). Bekdemir (2010) observed through qualitative interviews that the majority of pre-service teachers attributed characteristics of how the teacher behaved and taught as the primary sources of their mathematics anxiety.

Research has examined how mathematics anxiety impacts cognitive processes (Ashcraft et al., 2001). Math anxiety influences working memory capacity when an individual is engaged in mathematics procedures (Ashcraft et al., 2001). The more math anxious a person is, the lower their working memory capacity. This anxiety from simply thinking about doing a mathematics task brings on thoughts of worry, which can take up valuable working memory space, which in turn, limits working memory to complete the actual mathematics at hand (Ashcraft et al., 2001). Prior research has suggested many elementary pre-service educators possess mathematics anxiety (Bekdemir, 2010; Looney et al., 2017; Rayner et al., 2009). Math anxiety also has an impact on the courses students take and those who are math anxious tend to avoid or take fewer mathematics courses (Ashcraft et al., 2001). Future elementary educators may not only be entering teacher preparation programs with negative feelings and

anxiety towards mathematics, but as a result, they may also be entering with limited mathematical backgrounds compared to their peers.

The prevalence of mathematics anxiety among current and future educators has a number of different implications. Rayner et al. (2009) observed that pre-service educators with higher mathematics anxiety had lower performance on both procedural and conceptual mathematics assessments and suggested the increased anxiety may inhibit pre-service educators' development of mathematical content knowledge for teaching. Geist (2015) found that early childhood teachers' feelings and beliefs about mathematics had an impact on what they taught students and the level of importance they placed on mathematics within the curriculum. Math anxiety also influences educators' perceptions of their abilities, with those who are most anxious believing they are less able to competently engage with mathematics content (Geist, 2015). Evidence has suggested that mathematics anxiety can be transferred from teachers to students (Bekdemir, 2010), which heightens the importance of addressing the issue of mathematics anxiety in pre-service educators.

Development of Self-Efficacy. Bekdemir (2010) suggested teacher educators create opportunities for pre-service educators to self-reflect on their comfort with mathematics. It may be appropriate and worthwhile for teacher educators to provide strategies to cope with mathematics anxiety, to benefit the pre-service educators and also to use with future students (Bekdemir, 2010). Some research has suggested that coping strategies, such as relaxation, and a support system are effective in reducing mathematics anxiety in pre-service educators (Finlayson, 2014). Additional intervention strategies include narrative rehabilitation, which consists of examining

mathematical experiences through storytelling narratives, and bibliotherapy, which uses reading as a resource to promote change (Lutovac & Kaasila, 2014). Different approaches to teaching have also been linked to reducing mathematics anxiety (Finlayson, 2014). Finlayson (2014) argued that "setting up a constructivist learning environment where there is a spirit of inquiry, trust, and expectation, reduces math anxiety" (p. 111). Pre-service educators have noted that when mathematics is student-centric, builds off their prior knowledge, is collaborative, and is an active-learning process, their mathematics anxiety decreases (Finlayson, 2014). However, many pre-service educators have had prior experiences with mathematics education through more traditional mediums that are teacher-directed (Swars et al., 2007). It may be necessary to address these differences in teaching approaches to align with more research-supported best practices in mathematics education (NCTM, 2014). Pre-service educators also highlighted "the importance of having teachers believe in them and tell them they could do mathematics" (Finlayson, 2014, p. 111).

Several studies have further examined pre-service educator self-efficacy development through course interventions. Iossi (2007) discussed a variety of intervention techniques found within the literature that may reduce mathematics anxiety in post-secondary students. These techniques range from "curricular strategies, such as retesting, self-paced learning, distance education, single-sex classes, and anxiety courses" to "instructional strategies, such as manipulatives, technology, self-regulation techniques, and communication" to non-instructional, such as "relaxation therapy and psychological treatment" (Iossi, 2007, p. 30-31). Looney et al. (2017) examined mathematics anxiety reductions and changes in beliefs about mathematics

throughout a teacher preparation course. Looney et al. (2017) observed as the mathematics course progressed, participants reported increased confidence in their abilities and many students attributed this change to learning new instructional strategies. These preliminary findings indicate that the use of reflective strategies may facilitate change in beliefs, yet the authors noted that more research is needed to identify specific components within a course that develop self-efficacy and change beliefs (Looney et al., 2017).

Effective teaching requires a strategic introduction of mathematical content to students. NCTM (2014) highlighted that "developing conceptual fluency extends far beyond having students memorize facts or a series of steps unconnected to understanding. A rush to fluency, however, undermines students' confidence and interest in mathematics and is considered a cause of mathematics anxiety" (pp. 42-43). Stohlmann et al. (2015) explored changes in the beliefs and mathematical knowledge of pre-service educators during a course focused on representations of fractions. Results indicated beliefs about the importance of developing conceptual knowledge prior to procedures changed by the end of the course (Shohlman et al., 2015). Fraction activities helped pre-service educators integrate their procedural knowledge and see the importance of concepts. By the end of the course, the majority of participants believed it was important to learn concepts before procedures because students who demonstrate procedural understanding may not in fact understand concepts (Stohlman et al., 2015).

The use of manipulatives within teacher education programs as a mechanism to develop self-efficacy has also been explored (Althauser, 2018; Vinson, 2001).

Althauser (2018) found a course focused on inquiry-based instruction that utilized mathematics manipulatives influenced the self-efficacy of future educators. By the end of the course, observable overall gains in teaching efficacy and positive beliefs toward mathematics were present (Althauser, 2018). Future educators noted the course helped deepen their understanding of concepts and how to teach these to future students (Althauser, 2018). Vinson (2001) explored mathematics anxiety changes through a mathematics methods course that focused on concept development, concrete-pictorial-abstract, through the use of manipulatives. Overall, pre-service educator mathematics anxiety decreased, and several pre-service educators commented that manipulatives and pictorial representations helped their mathematical understanding (Vinson, 2001). However, some pre-service educators experienced increased anxiety and attributed this to the use of manipulatives for the first time. A combination of relearning mathematics and learning to use new tools caused an anxiety increase (Vinson, 2001). This finding suggests there may not be a universal instructional strategy or intervention which benefits self-efficacy development for all students and that different strategies influence pre-service educators differently.

Swars et al. (2007) observed increases in the self-efficacy of pre-service teachers and attributes these changes to what Bandura (1977) indicated as integral mediums of self-efficacy development: knowledge experiences and vicarious experiences. The pre-service educators in the Swars et al. (2007) study were able to practice and master their mathematical teaching skills through extensive field experience opportunities. Additionally, constructivist teaching strategies were demonstrated by teacher educators. Swars et al. (2007) found constructivist views

were adopted and embraced by pre-service educators even when they were engaged in the field work and were presumed to encounter more traditionally held mathematical views and teaching styles of veteran teachers. Swar et al. (2007) speculated this may be due to the fact that these pre-service educators were exposed to an extended, two-semester mathematics methods course and field experience program, which may have facilitated the development of self-efficacy.

Gibbons et al. (2018) argued the importance of pre-service educators "developing productive mathematical identities" (p. 21). Mathematical identity is "one's knowledge, habits, beliefs, values, and commitments as it relates to one's participation within a community of practice" (Gibbons et al., 2018, p. 21). Within teacher preparation, it has also been defined as "narratives pre-service teachers tell themselves or others about themselves as mathematics teachers and learners" (Lutova & Kaasila, 2014, p. 130-131). A component of mathematical identity is "mathematical agency" or "students' capacity to think of themselves as doers of mathematics who can make progress on challenging issues, trust in the conclusions that they draw, create arguments and explanations, share their own ideas, and extend the ideas of others" (Gibbons et al., 2018, p. 22). A secondary component of mathematical identity is "mathematical authority" which consists of collaborative math work with classmates (Gibbons et al., 2018).

Gibbons et al. (2018) examined how facilitative strategies of teacher educators influence the mathematical identities of pre-service educators. Findings indicated significant shifts in teachers' confidence in reasoning and engaging in mathematics discussions by the conclusion of the course (Gibbons et al., 2018). The authors

suggested facilitation practices may have had a positive influence on this change, yet these practices cannot be definitely linked to mathematics identity development due to difficulties in measuring the pre-service educator's perception of themselves at the specific time of the instructional facilitation (Gibbons et al., 2018). However, further research into this area may provide additional insights into how pre-service educators can strengthen their self-images as capable mathematics learners and educators.

Additional examination of mathematical identity includes a study by Lutovac and Kaasila (2014), in which they utilized a narrative framework to explore future-focused math identities of pre-service educators. Narrative mathematics identity is fluid and changes as an individual engages in reflection (Lutovac & Kaasila, 2014). This construction and reconstruction of mathematical identity suggest teacher education programs may be in a position to enact change in the mathematical identities of future educators. It is not only important for future educators to disentangle their own past experiences in mathematics, but they must also develop productive identities for their future work as teachers of mathematics. Lutovac and Kaasila (2014) observed two different types of future mathematical identities, "decisive cases" and "irresolute cases" (p. 139). Decisive cases had a balance of, on the one hand, fears and negative views of mathematics, and positive aspirations which indicated motivation, on the other hand (Lutovac & Kaasila, 2014). These decisive cases were observed in environments in which pre-service teachers were encouraged to focus on identity work and had an opportunity to understand their past experiences in a manner that allowed them to focus on the development of their future selves (Lutovac & Kaasila, 2014). Conversely, the irresolute cases were less balanced, less goal-oriented, and submitted

to their mathematical fears, and were observed in environments in which identity work was not emphasized in mathematics teacher education coursework (Lutovac & Kaasila, 2014). As Gibbons et al. (2018) also have suggested, it appears mathematical agency may play an integral role in positive mathematical identity development (Lutovac & Kaasila, 2014). These studies provide insights; however, the sample sizes were small and additional research must further investigate both specific relationships between intervention techniques and mathematical identity, and the generalization of these findings to broader populations.

Mathematical identity development can be considered from various aforementioned theoretical frameworks. For example, from adult learning theories and transformational learning theory, in particular, mathematical identity work can be viewed as a communicative mechanism in which frames of reference may be able to change or evolve (Lutovac & Kaasila, 2014; Mezirow, 1997). Whether a course or courses during a teacher education program have the potential to create sufficient lasting changes to one's frame of reference in regard to mathematics and mathematical abilities, remains largely unknown. Specifically, what are the most effective intervention strategies that can be readily adopted within teacher education programs, that may not only develop mathematics self-efficacy but perhaps dually benefit the development of mathematical knowledge for teaching? These preliminary identity studies suggest there is a need for further investigation into how mathematical identities develop throughout pre-service programs.

In relation to Bandura's (1977) self-efficacy theory, mathematical identity work may have a positive influence on the self-efficacy of future educators. For

example, narrative identity work could arguably influence an individual's emotional arousal in that they come to understand their current situation in relation to mathematics is better than they previously thought (Bandura, 1977). This thinking could potentially lessen the emotional arousal associated with mathematical tasks (Bandura, 1977). However, cognitive processes and self-efficacy development are complex and multidimensional, and Bandura (1977) indicated a change in self-efficacy may not occur if an individual's lived experiences are deeply ingrained and counter-intuitive to efficacy expectations. This begs the question, to what extent can courses create change in those who have deeply rooted negative experiences in mathematics?

Elementary Educators' Beliefs Related to Mathematics. Taking into consideration pre-service educators' mindsets and attitudes can also provide insights into views of mathematics. Ire et al. (2018) investigated teacher mindsets of pre-service English Foreign Language teachers and found the majority of teacher candidates had a growth-oriented view toward learning. However, when examined more closely, these future educators viewed certain interpersonal characteristics as less malleable in terms of growth potential compared to the more technical or pedagogical skills needed for teaching, such as classroom management that were viewed as malleable (Ire et al., 2018). The authors suggested these technical pedagogical skills may have been viewed as more malleable because they were explicitly taught within the teacher preparation program (Ire et al., 2018). The implications of these findings suggest emphasizing the malleability of mathematical knowledge for teaching may enhance educator confidence. If pre-service educators come to view their knowledge

as fluid rather than fixed, they may more readily embrace their own mathematics learning development.

Although evidence has suggested pre-service educators have limited knowledge and may not be overly confident in or enjoy mathematics, some research has suggested pre-service educators recognize the importance of mathematics and have good intentions to incorporate STEM learning within their classrooms (Kurup et al., 2019). Pre-service educators who have experienced mathematics reform efforts have been found to demonstrate the importance of mathematics within everyday life contexts, compared to more veteran educators who experienced more traditional mathematics education (Leavy et al., 2017).

Beliefs in Relation to Students and Efficacy Implication for Students. Bates et al. (2011) explored mathematical self-efficacy and mathematical teacher efficacy of pre-service educators and observed a relationship between self-efficacy and mathematics achievement. Findings indicated teacher confidence positively influenced mathematics teaching efficacy. Bates et al. (2011) found a positive correlation between mathematics self-efficacy and mathematics performance of pre-service educators. The better pre-service educators performed on the basic mathematics assessment, the more confident in their mathematical abilities and abilities to teach mathematics. Only the pre-service teachers who were the most self-efficacious believed they had an influence over the mathematics performance of their students (Bates et al., 2011). Bates et al. (2011) suggested opportunities within teacher education for self-efficacy in mathematics to be developed.

Educator attitudes toward mathematics and educator mathematics self-efficacy have a number of implications for students. As Geist (2015) indicated, those educators with a greater level of mathematics confidence are more likely to place increased importance on mathematics as a subject and work to incorporate developmentally appropriate mathematics tasks within their instruction. Chang (2015) examined the effects of fifth-grade mathematics teachers' efficacy on student mathematics self-efficacy and mathematics achievement. Results indicated mathematics teacher efficacy is a significant predictor of student academic outcomes (Chang, 2015). Mathematics teacher efficacy also had a significant impact on student mathematics efficacy. The higher level of mathematics self-efficacy a teacher had, the higher the students' mathematics self-efficacy. As a consequence, the higher the self-efficacy of the student, the better the student performed academically (Chang, 2015). Similarly, Looney et al. (2017) highlighted the influence educators' attitudes towards mathematics can have on students, saying, "helping students to get excited about mathematics is likely to fail, if they themselves (as teachers) are not excited about it" (p. 35). Considering the influences educators' attitudes and self-efficacy can have on their instruction and students, it becomes crucial to examine the mathematics self-efficacy of pre-service educators who will soon be impacting the next generation of mathematics learners.

Not only does teacher content knowledge impact students' mathematics development, but so does a teacher's belief in their own mathematical abilities. As indicated by these studies, mathematics anxiety and negative views about mathematics abilities and mathematics instruction are common in pre-service elementary educators

(Geist, 2015). Beliefs and confidence can influence a great deal in the mathematics classroom. It can impact even subconsciously how teachers prioritize mathematics learning, how they view their learners' mathematical abilities, and how they believe mathematics is best taught and learned. All these components can impact students in different ways, including how they view themselves as mathematics learners, their actual academic achievements, and future trajectories that do or do not incorporate mathematics as a foundational component. Teacher education has an opportunity to employ instructional strategies to strengthen confidence in teachers' mathematical abilities and their confidence in teaching mathematics, as well as provide strategies and opportunities for educators to examine their own past experiences with mathematics and mathematics learning.

Issues with Self-Efficacy Instruments

More research is needed to understand how pre-service teachers are perceiving their own development of self-efficacy and what factors they identify as crucial to building their capacities as mathematics learners and teachers. Many existing studies on teacher self-efficacy utilize self-efficacy instruments, and the validity of some of these instruments has been questioned. Some instruments, such as Gibson and Dembo's (1984), Teacher Efficacy Scale (TES), claim to measure self-efficacy, but actually measure participant outcome expectancy, which is not a component of self-efficacy, but rather locus of control (McGee & Wang, 2014). Additionally, there are complexities in measuring self-efficacy within specific content areas. The Mathematics Teaching Efficacy Beliefs Instrument (MTEBI) has been used to measure teaching self-efficacy in mathematics, however, this instrument measures

both beliefs teachers have in themselves to teach mathematics (perceived mathematics teaching self-efficacy) and belief in their ability to produce desired outcomes as a result of their teaching (mathematics teaching outcome expectancy) (Mcgee & Wang, 2014). Bandura (2006) distinguished perceived self-efficacy from outcome expectancies by saying, "perceived self-efficacy is a judgment of capability to execute given types of performance; outcome expectations are judgments about the outcome that are likely to flow from such performances" (p. 309). This is not to say beliefs related to outcome expectancy and the construct of locus of control are not important variables in understanding teacher and pre-service teacher development. However, it is concerning that many published studies have used instruments and made claims regarding self-efficacy, when in fact the instrument may be measuring another construct.

In their own study to determine validity around the Self-Efficacy for Teaching Mathematics Instrument (SETMI), Mcgee and Wang (2014) stated, "Through the process of examining structural validity, it became clear that there is potential for self-efficacy to be much more complex than was initially thought" (p. 400). Mcgee and Wang (2014) further discussed the need to consider mathematics content knowledge and the personal characteristics of teachers, as these are likely important factors that influence teachers' self-efficacy. These concerns around the validity of existing instruments and questions surrounding influential factors and complexities of mathematics teaching self-efficacy, create a need to explore how teachers and pre-service teachers understand their own development as learners and teachers of mathematics. It is possible that existing instruments may not be capturing fundamental

variables of pre-service teachers' mathematics self-efficacy and mathematics teaching self-efficacy.

Math Experiences and Opportunities to Learn within Teacher Education

Tatto & Senk (2011) described the first large-scale international study that compared teacher education in mathematics, the Teacher Education and Development Study in Mathematics (TEDS-M). The TEDS-M was a quantitative study conducted in 17 countries that examined mathematical knowledge for teaching, encompassing both content knowledge and pedagogical content knowledge; opportunities to learn mathematics; and beliefs about mathematics, teaching, and learning (Tatto & Senk, 2011). Main findings suggested there is great variation in opportunities to learn and mathematical knowledge for teaching between and within countries. Furthermore, there were large variations in mathematical content knowledge and mathematical pedagogical content knowledge across programs and within each country (Senk et al., 2012).

TEDS-M highlighted that there was a range of mathematical knowledge within the same country (Senk et al., 2012). In the United States, future primary teachers did well in reaching an initial anchor point domain for mathematical knowledge, but only 50% reached the second anchor point, indicating a lack of deep understanding and ability to engage in mathematics. For example, future educators who reached anchor point 1 could perform simple whole number calculations, but had more trouble applying concepts to problem solving as “they tended to overgeneralize and had difficulty solving abstract problems and those requiring multiple steps” (Tatto & Senk, 2011, p. 128). Those who reached anchor point 2 “generally did well on items testing

‘knowing’ and on standard problems about numbers, geometry, and algebra, classified as ‘applying,’ but they had more difficulty answering problems that require more complex reasoning in applied or non-routine situations” (Tatto & Senk, 2011, p. 129). Tatto and Senk (2011) suggested a need to further examine anchor points for mathematical content knowledge and mathematical pedagogical knowledge within countries and look more closely at the curriculum of institutions that top scorers attended.

In response to the TEDS-M research, Canadas et al. (2013) examined structural differences in Spain’s teacher education programs by analyzing syllabi from participating schools. An important distinction between Spain and the United States is that in Spain there are national guidelines for all teacher education programs with 120 of the 190 program credits dictated by this set of common guidelines (Canada et al., 2013). Even given most of the program credits are mandated, differences were observed within institutions teaching the same course. The study also found a great emphasis on general pedagogical knowledge compared to mathematical pedagogical content knowledge (Canadas et al., 2013). Given the widespread variation in programs within a country that has national guidelines for curriculum, it brings into question the type of variation that might be observed when examining programs in the United States.

Qian and Young (2016) examined TEDS-M data from five countries, including the United States to better understand the mathematical content knowledge and mathematical pedagogical content knowledge of pre-service educators based on differences within their teacher education programs. In contrast to other countries in

the study, the United States' teacher preparation program components were not strongly correlated to pre-service educator mathematical content knowledge or mathematical pedagogical content knowledge (Qian & Young, 2016). The study found that pre-service educators' "mathematical content knowledge and mathematical pedagogical content knowledge mostly depends on their previous mathematics achievement and their parents' education and their experiences in teacher preparation have little effect" (Qian & Young, 2016, p. 393).

Qian and Young (2016) also found that in the United States the greater the number of mathematics content courses an institution of higher education required, the lower the scores of mathematical content knowledge were. The authors further observed a difference between countries in the number of mathematics content courses required, with the United States requiring more than other countries. The authors suggested perhaps incoming pre-service educators were not as prepared in mathematics compared to other countries that had applicants that entered with stronger foundations in mathematics (Qian & Young, 2016). These findings are concerning, indicating that pre-service coursework in the United States may not adequately prepare future educators to teach mathematics effectively.

Cohen and Berlin (2020) explored "opportunities to learn" (OTL) pulling from Grossman et. al (2009)'s pedagogies of practice framework in pre-service coursework. Cohen and Berlin (2020) wanted to explore within-program variation in teacher education programs in the United States and argued differences could be a result of individual characteristics and perspectives. Cohen and Berlin (2020) described that:

> The construct of OTL is thus based on how a particular setting facilitates a candidate's development as a member of the learning community over time. The opportunity for learning is influenced by the learner, the nature of the task, and the tools the teacher educator employs. (p. 435)

OTL as a construct is complex and has been interpreted and measured throughout the literature in different ways (Cohen & Berlin, 2020). Cohen and Berlin (2020) discussed the challenges in measuring OTL and that many studies rely on curricular reviews that assume students learned intended learning objects, rather than obtaining an understanding of how the curriculum is enacted and what learning actually takes place. Cohen and Berlin (2020) described this difference as the intended opportunity to learn compared to the enacted opportunity to learn. Intended opportunities to learn can be understood through a review of the course program descriptions and syllabi, but to better understand the enacted curriculum, an investigation of curriculum materials and observations is warranted (Cohen & Berlin, 2020). Using Grossman's (2009) pedagogies of practice framework, Cohen and Berlin (2020) created a survey to measure pre-service educators' experiences with representations, decompositions, and approximations of practice. Although these pedagogies of practice have been defined and identified in different ways throughout the literature, Cohen and Berlin (2020) broadly described them as follows:

> Representations include videos, student work, and lesson plans. Decompositions involve the taking apart and analyzing representations. A teacher educator might annotate (decompose) a lesson plan (representation) to highlight salient aspects. Approximations--role plays, rehearsals, simulations--

> afford opportunities to attempt practice that candidates will likely use as teachers. (p. 439)

Cohen and Berlin (2020) found greater within-program variation for all types of opportunities to learn than comparing different programs between institutions. They believed differences were not due to an actual difference in learning experiences, but due to student perceptions of those experiences (Cohen & Berlin, 2020). Norton and Allen (2020) argued the point that it is critical to understand pre-service educators' views of learning and highlighted these future educators bring their own set of experiences as mathematics learners from at least 12 years of education leading up to their teacher education programs. Consideration of pre-service educators' readiness to learn could influence how they engage with the intended curriculum and the growth they have throughout their teacher education program.

Norton and Allen (2020) found that most pre-service educators wanted to gain mathematical content knowledge and mathematics-specific pedagogy, and results indicated pre-service educators valued these opportunities. A particularly interesting finding from Norton and Allen's (2020) study was that within one of the cohorts, pre-service educators responded negatively and were resistant to the high expectations that they would be able to master the mathematical content within a short period of time. Norton and Allen (2020) cautioned that these negative reactions to curriculum and high expectations could in fact damage pre-service educators' self-efficacy.

Ghousseini and Herbst (2016) also examined pedagogies of practice and opportunities to learn mathematics and specifically focused on mathematics discussions. Ghousseini and Herbst (2016) provided examples of the benefits to pre-

service educators engaging in mathematical discussions, suggesting it provided them a chance to examine, challenge, and grow their own level of mathematical understanding in an authentic manner. However, one interesting question for consideration was how do students respond when their points are refuted or questioned during mathematical discussions and how does this influence their mathematics self-efficacy? In the study, a student's idea was questioned by the class, and "she expressed jokingly her feeling of incompetence, hence raising concerns for some PSTs about the affective risks entailed in the process of public refutation and validation" (Ghousseini & Herbst, 2016, p.95). This study was conducted with secondary mathematics education majors, who arguably have more interest and experience with mathematics, and likely a lower level of mathematics anxiety compared to their elementary counterparts. Providing meaningful opportunities to learn can be impactful to pre-service educators, but a closer examination of how these experiences are perceived at the individual level is necessary.

This examination of a pre-service educator's readiness to learn may also have implications on a teacher educator's instructional style. Knowles' (1986) adult learning theory, suggested the importance of student-directed learning experiences; however, as mentioned previously, some adult learners seek more guidance (Merriam, 2001). Kirschner et al. (2006) argued that constructivist approaches that rely on minimal teacher guidance are likely to be ineffective if students do not have well-developed prior knowledge to guide them. These studies further emphasize the need to consider pre-service educators' experiences and perspectives to create a productive mathematical learning environment.

Opportunities to learn may be complex and it is important to understand how they are perceived by students. Cohen and Berlin (2020) suggested more research is needed to understand these perceptions of opportunities to learn and pointed to a need for more mixed-methods research in this area. A limitation of their study was the methodology; the survey instrument did not provide detailed examples of what is considered a particular type of opportunity to learn, allowing for errors in omissions or incorrect categorization (Cohen & Berlin, 2020). Additionally, the study used the intended curriculum through a review of syllabi rather than direct observations of learning opportunities (Cohen & Berlin, 2020), which would have provided a richer and perhaps more accurate depiction of pedagogies of practice. As Cohen and Berlin (2020) have highlighted the importance of understanding individual perspectives, as well as how these influence an individual's learning experience, it is also important to consider self-efficacy within this context. It may not be enough for teacher education programs to provide meaningful learning opportunities such as representations, decompositions, and approximations, without considering how these learning experiences will be received by the pre-service educators and even enacted by the teacher educators.

Stakeholder and Organization Data

There is a growing economic need for a well-trained STEM workforce with a strong foundation in mathematics (CoSTEM, 2018; Fayer et al., 2017). Even with curricular efforts and reforms over the past several decades, our students in the United States are underprepared, scoring, on average, less on mathematics achievement measures than their international counterparts (CoSTEM, 2018). The 2018 CoSTEM

report, highlighting data from 2005-2015, also indicated less than 25% of prospective college students are prepared for courses required of STEM majors. More specific to teacher education, TEDS-M data indicated United States pre-service elementary educators lack a depth of knowledge in mathematics content overall (Tatto & Senk, 2011). Local state data (NHDOE, 2019) indicate that there is a 57.72% post-secondary enrollment rate in the state. Recent SAT data from 2018 indicate 49% of the state's SAT test-takers met the mathematics benchmark (College Board, 2018). These statistics are important for teacher preparation programs to review to bring awareness and consideration to the mathematics learning readiness of their teacher educator candidates and to work towards better preparing the next generation of mathematics learners.

Local Department of Education Mathematics Proficiency Data

Local student mathematics achievement data can provide a glimpse into mathematics learning within the state. Recent data from the 2018-2019 academic year indicates the overall mathematics achievement rate in the state is 48% (NHDOE, 2019). This mathematics achievement has remained stagnant at 48% between 2017-2019. It is interesting to examine mathematics achievement proficiency over time. In looking at data from the 2018-2019 academic year, 57% of third graders are proficient in mathematics. By fourth grade, there is a decrease to 52% proficiency, and then proficiency fluctuates between mid-low 40% in grades six, seven, eight, and 11 (NHDOE, 2019). In one regard it could be argued it is reasonable to anticipate that as students progress in their education and interact with more advanced mathematics, there may be a reduction in achievement. However, this conclusion negates the fluid

capabilities of learners and assumes a deficit-view of students. Instead, the usefulness of this data is to indicate the need to better understand the mathematical knowledge and experiences of these learners. It is possible that gaps in understanding exist before third grade and only become exacerbated as content builds on prior concepts. This would call for the examination of those not meeting proficiency from the start, and also of students who have demonstrated proficiency at earlier grade levels only to later fall behind.

This local achievement information can provide additional insights such as mathematics proficiency for various populations of students. For economically disadvantaged students there are stark differences in achievement. The largest percentage of these students fall into the lowest level of mathematics achievement in the state, at 39%, followed by the next largest percentage of 32% in the second lowest level of math achievement range (NHDOE, 2019). This means 71% of the state's economically disadvantaged population is below proficient in mathematics. The achievement disparities continue for other marginalized groups who are below proficiency in mathematics; 76% of English Language Learners, 82% of students with learning disabilities, 65% of students in foster care, 77% of students who are homeless, 70% of migrant students, 65% of American Indian or Alaskan students, 75% of black or African American students, and 69% of Hispanic or Latino students. This is compared to 51% of white students below proficiency. The overall low mathematics proficiency level is concerning, but of even greater concern is the inequity made apparent in this data.

A final local consideration is the curricular variations in teacher preparation programs within the state. There are 12 State Board-approved professional educator preparation programs, and 11 of these 12 offer a program to become an elementary educator (NHDOE, 2020). These programs vary in the number of mathematics preparation courses required and offered, as well as the nature and content of these courses. Research at the local level can provide insights into the types of opportunities to learn and the impact these different experiences have on future educators' mathematical knowledge for teaching and the development of mathematics self-efficacy.

Summary

Mathematics education reform and research is not a new topic, but there remain unanswered questions, and further work in this area is necessary to make profound changes to impact the current and next generations of students. Studies have provided insights into the importance of different types of mathematics knowledge beyond understanding content and procedural steps (Ball et al., 2008; Hill et al., 2005; Shulman, 1986), yet many intricacies of these knowledge subsets and mechanisms to effectively develop them within teacher education preparation remain unknown (Cohen & Berlin, 2020; Tatto & Senk, 2011). Research has also pointed to a prevalence of mathematics anxiety in pre-service educators (Bekdemir, 2010; Geist, 2015; Looney et al., 2017) and limited mathematics knowledge, especially conceptual knowledge of later elementary school content (Norton, 2017; Van Steenbrugge et al., 2014). Both low self-efficacy and limited teacher understanding of mathematical concepts have negative influences on students (Chang, 2015; Geist, 2015; Krawec &

Montague, 2014; Van Steenbrugge et al., 2014). Teacher education preparation is an opportunity to address concerns in mathematics beliefs and knowledge. It provides a supportive environment to strengthen skills. Many studies have explored mathematics teacher preparation, and some have examined specific instructional strategies that influence positive beliefs and knowledge improvement (Althauser, 2017; Depaepe et al., 2018; Gibbons et al., 2018; Looney et al., 2017; Lutovac & Kaasila, 2014; Reid & Reid, 2017; Strawhecker, 2005; Stohlmann et al., 2015) However, these studies leave unanswered questions, specifically regarding how pre-service educators perceive the factors that facilitate their development as mathematics learners and teachers.

This study began to explore these gaps and the findings can be used to inform elementary teacher education programs. Specifically, this study explored the experiences and conditions that pre-service teachers and new in-service teachers view as influential in developing their capacities to learn and teach elementary mathematics.

Chapter Three: Methods and Design for Action

This research examined how elementary pre-service and new in-service teachers perceived their development of mathematics learning and teaching as they become elementary educators. This study explored the experiences, relationships, and conditions that pre-service teachers and new in-service teachers view as influential in developing their capacities to learn and teach elementary mathematics.

Central Research Question

1) How do emerging elementary teachers perceive the experiences and conditions, and relationships that influence their development as learners and teachers of mathematics?

This chapter presents the purpose of the study and outlines the research design. A rationale for the research design is considered. Next, the chapter includes a description of participants, recruitment, and methods for collecting data. The next section outlines proposed methods for analyzing the data. This is followed by a research timeline from IRB approval through data analysis. Lastly, this chapter addresses the limitations of the research study and design.

Study Purpose and Design

Purpose

The purpose of this study was to examine how emerging elementary education professionals understand their own capacities to do and teach elementary mathematics and to identify factors that influence their development as teachers of elementary mathematics. This research focused on the experiences and beliefs of the participants and how these perspectives and beliefs influence pre-service and new in-service

elementary teachers as they prepare to enter their field. Specifically, this research explored how pre-service and novice in-service elementary teachers understand their own evolving capacity to do and teach mathematics. This study will provide insights into how prospective and beginning elementary teachers develop their mathematics self-efficacy, mathematics teaching self-efficacy, and develop their teaching skills and mathematical knowledge.

Worldview and Reflexive Statement

I come to this research with my own experiences in mathematics and mathematics education. I was both a student and an elementary teacher of mathematics. I came to this research with knowledge of mathematical concepts and a passion for the subject. However, I also experienced my own self-doubts and moments of questioning my capabilities. As a former elementary educator, I frequently spoke with colleagues who did not enjoy mathematics or who did not feel confident in mathematical concepts. These experiences led me to this research to better understand the mathematics experiences of elementary educators.

In approaching this research design, I considered my own system of belief. This path was not linear. My background in the sciences initially had me considering a design that centered on quantitative data, looking for correlations and relationships in pre-identified variables. However, the more I came to understand the complexities of mathematical content knowledge and the knowledge needed to become a teacher, the more I began to doubt the value of using solely quantitative methods for my own study. As I reviewed the literature, I began to see limitations and unanswered questions about how pre-service teachers internalize and understand their own

development as mathematics learners and teachers. I realized my study and research question would not be answered adequately using quantitative methods, and instead qualitative methods would be most appropriate. In reflecting on my own belief system, I discovered my worldview is in alignment with social constructivism. Social constructivists believe in a subjective reality that is grounded in individuals' experiences and how they interpret those experiences (Creswell & Creswell, 2018). The purpose of this study was to do just that, understand the realities of emerging elementary teachers on their paths to becoming teachers of mathematics.

Rationale

This Dissertation in Practice (DiP) was an exploration of participants' mathematical experiences. The findings of this study are intended to inform future action and practice within elementary education teacher education. As this research focused on the experiences and perceptions of participants and the complexities of how individuals come to view their development as teachers, a qualitative approach was appropriate. This research study took a narrative inquiry approach to explore the perspective of emerging elementary mathematics educators with the intention of providing practice recommendations.

In describing narrative inquiry, Clandinin (2006) stated, "it is commonplace to note that human beings both live and tell stories about their living. These lived and told stories and talk about those stories are ways we create meaning in our lives as well as ways we enlist each other's help in building our lives and communities (p. 44). Clandinin further stated that "individual experiences are shaped by the larger social, cultural, and institutional narrative within which they live and have lived." Through

narrative inquiry researchers come to understand individual experiences as they are influenced by and exist in different contexts.

Clandinin (2006) describes the "three-dimensional narrative inquiry space" as "the personal and social (interaction)," "past, present and future (continuity)," and "place (situation)" (p. 47). These three dimensions provide a holistic understanding of the experiences and the meaning of those experiences to the individuals. The narrative inquirer engages participants within each of the three dimensions and collects field texts (Clandinin, 2006). The aforementioned philosophical underpinnings of narrative inquiry, as established in the literature, are the foundation and guide for this research methodology. Narrative inquiry was chosen as a methodological approach to elicit and explore the perspectives of participants.

Ethical Approval, Context, and Participants

Participants were recruited and selected in a purposeful way given the need to understand the unique perspectives of a specific population. The target population of participants includes pre-service elementary teachers who completed a State Board Approved Teacher Education program and beginning elementary educators in their first three years of teaching in a northeastern state. Through collaboration with teacher educators and elementary program coordinators, pre-service elementary education participants were identified and emailed. Through professional contacts and requests sent to elementary public school administrators, in-service elementary educators in their first one to three years of teaching were emailed the recruitment materials. Eligible participants were given recruitment information about the purpose of the

study and the time commitment to participate. Interested participants were provided an electronic copy of the informed consent form to review and sign.

Figure 3.1

Targets and Timeline

Table 3.1

Pre-Service Teacher Participants

Participant (pseudonyms)	**Participant Description**
Brandi	• Graduating with a master's degree in elementary education and applying for classroom teaching positions. • Completed bachelor's degree in human development and family studies.
Chloe	• Graduating with a bachelor's degree and has been offered a job as a third-grade teacher for the following school year.
Heather	• Graduating with a bachelor's degree in elementary education and has applied for a master's in a special education program at the same institution.
Naomi	• Completing a master's degree in elementary education. • Completed bachelor's degree in human development and family studies. • Has completed all coursework and is looking forward to completing her full-year student teaching in an urban setting.
Sarah	• Graduating with a bachelor's degree in elementary education and applying for classroom teaching positions. • Non-traditional students who returned to school after starting her family.

The first group of participants in this study consisted of five pre-service elementary school teachers enrolled in a State Board Approved Professional Educator Preparation Program in a northeastern state. Pre-service elementary participants were recruited from four public state institutions within the state. Three institutions were traditional residential campuses. Two of which offer four-year bachelor's degrees in Elementary Education which lead to teacher certification. The third residential institution offers a Master of Education degree which leads to teacher certification. The final institution is a non-residential campus that offers a post-baccalaureate teacher certification.

Participants were recruited from the Elementary Education programs at the aforementioned institutions. Participants were recruited to participate in the study via email communication. Program coordinators from the teacher education programs identified eligible participants to be sent recruitment information. The email sent to participants included information about the study, including the purpose, and scope of participation. The email communication also included a link that directed interested participants to the informed consent form. All eligible participants were from college-aged populations and consisted of participants who were 18 years or older and English speakers.

Out of the five participants who participated in the research study, three out of the four institutions were represented. There were no participants from the non-residential campus. Of the participants, four out of five were in the final semester of their teacher education programs and preparing to graduate. One of these participants had already received a job offer for the following school year, two more were actively

searching for positions, and one had applied to a master's program in Special Education. The last pre-service teacher had completed all her coursework for her program but needed to complete a full year of student teaching during the next school year.

Table 3.2

In-Service Teacher Participants

Participants (pseudonyms)	**Participant Description**
Amelia	• First-grade teacher with one year of classroom teaching experience. • Completed an undergraduate degree in education, but "didn't feel ready" and enrolled in a master's program with a dual specialty in special education. • Began teaching during the covid-19 pandemic
Jon	• Third-grade teacher with one year of classroom teaching experience. • Completed a bachelor's degree in elementary education • Began teaching during the covid-19 pandemic.
Meghan	• Completed bachelor's degree in elementary education • Non-traditional student with a background in business. Spent years gaining experience substitute teaching. • Has taught multiple grades (third, fourth) • Entering third year as a classroom teacher
Tara	• Completed bachelor's degree in elementary education • Non-traditional student with a young family. • Has taught multiple grades at a private school • Entering third year as a classroom teacher

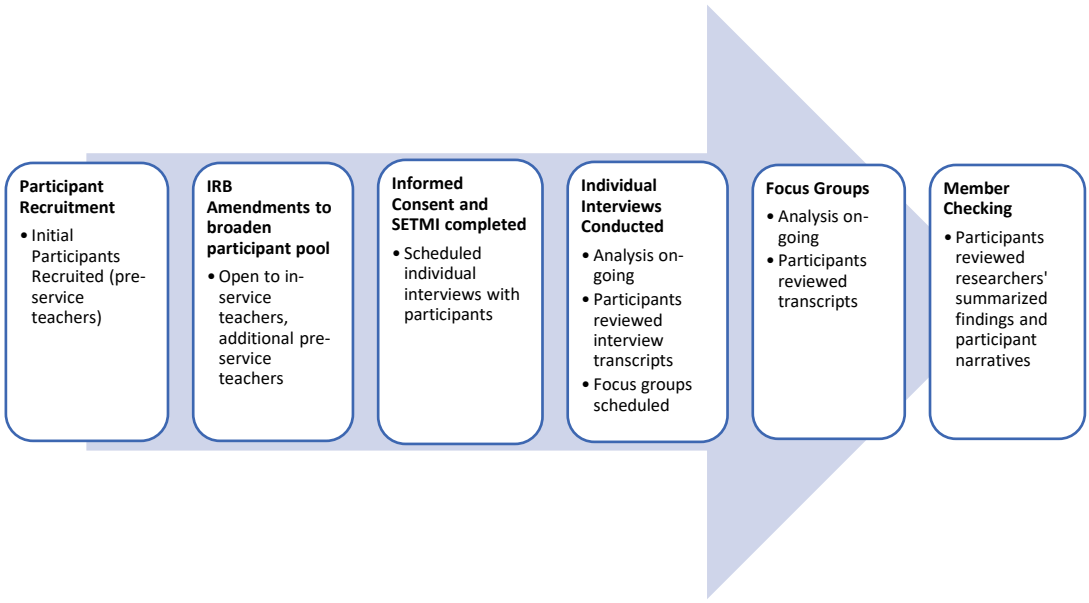
The second group of participants for this research study included four new in-service elementary teachers within their first three years of teaching at an elementary school in a northeastern state. These participants were also recruited via email communication. The email sent to in-service participants contained information about the study, including the purpose, and scope of participation. The email communication also included a link that directed interested and eligible participants to the informed consent form. Participants from this group included a first-grade teacher with two

years of classroom teaching experience, a third-grade teacher with one year of classroom teaching experience, and two teachers entering their third year of teaching who had taught at multiple grade levels.

Data Collection and Specific Practices

Figure 3.2

Data Collection Process



The central research question of this study is: How do pre-service and in-service teachers perceive the experiences and conditions that influence their development as learners and teachers of mathematics? To answer this exploratory question, several methods were used to gather participants' perspectives and narratives. Methods used to gather data from participants included a self-efficacy instrument, focus group interviews, and individual interviews. The Self-Efficacy Teaching for Mathematics Instrument (SETMI; McGee & Wang, 2014) was administered electronically prior to participating in an individual interview. The SETMI is a 22-item instrument that uses a five-point Likert-based scale. The SETMI

has been found to be a valid and reliable tool to measure two subsets of self-efficacy: pedagogy in mathematics and teaching mathematics content with reliability Cronbach alpha scores of .86 and .93, respectively (McGee & Wang, 2014).

Hour-long individual interviews were scheduled with participants to meet their scheduling needs. Five elementary pre-service teachers described previously participated in individual interviews. Four in-service teachers in their first three years of teaching, as described previously, participated in individual interviews. An hour-long virtual focus group was conducted with the group of pre-service teachers following individual interviews. This focus group consisted of four of the five pre-service teachers from two public institutions in the state. One participant was unable to attend the focus group due to a family emergency. A separate hour-long virtual focus group was conducted with the in-service teacher participants following their virtual individual interviews. The combination of focus groups and interviews allowed for narratives to emerge around a shared experience and in a shared environment (during focus groups), as well as allowed for a more personalized understanding of the meaning and significance of individual experiences (during individual interviews).

Standardized open-ended interviews were conducted as described by Patton (2002). These hour-long interviews were scheduled at times and locations that were convenient for the participants, such as public spaces on or near the participants' campuses or scheduled virtually with in-service teacher participants. Prior to beginning each individual interview, the researcher reviewed the informed consent form and obtained verbal permission to record the interview session using a SONY ICD-UX570 stereo digital recorder device. Detailed field notes were kept throughout

the session. Additionally, the researcher wrote up an analytical memo detailing her general sense of the conversation and the individual's story. These memos were kept within the researcher's field journal and used as part of the analysis process. After each session, the field journal was secured in the researcher's locked filing cabinet in her home office. The individual interviews followed an established individual interview protocol created by the researcher (see Appendix E). During the interview, the researcher engaged participants to understand their experiences within the "three-dimensional narrative inquiry space" (Clandinin, 2006, p. 47) by thoughtfully asking questions that elicited participants' understanding of their situational contexts, social interactions, and mathematics understanding and development over time.

The two focus groups, pre-service participants and in-service participants, were conducted as described by Krueger (1994) and followed an established focus group protocol. Krueger (1994) indicated focus group interviews are appropriate for exploratory studies and also when the "purpose is to uncover factors relating to complex behavior or motivation" and "provide insight into complicated topics" (p. 45). A focus group protocol was followed with a series of pre-identified open-ended questions (see Appendix D). These questions were structured following Krueger's (1994) question types: "opening question," "introductory questions," "transition questions," "key questions," and "ending questions" (pp. 54-55). The focus group was structured in a strategic way as recommended by Krueger (1994). Participants were initially welcomed to the focus group and the informed consent form was reviewed. Participants were provided the opportunity to ask any questions regarding the informed consent at this time. Then the purpose of the research and focus group was

discussed. Prior to beginning questions, ground rules regarding confidentiality were established. With verbal permission from participants, the focus group session was recorded using a SONY ICD-UX570 stereo digital recorder device. Detailed field notes and analytical memos detailing general impressions were compiled in the researcher's field journal. After the focus group session, the field journal was secured in the researcher's locked filing cabinet in her home office. The field notes and analytic memos were used as part of the analysis process. An interactive brainstorming activity was used to create a word cloud of participants' responses in the focus group (see Figures 4.2 and 4.3).

Data Analysis and Evaluation

As shown in Figure 3.3, data were analyzed using a narrative thematic analysis approach (Butina, 2015). The overall analysis was guided by the six phases of reflexive thematic analysis (Braun & Clarke, 2022). Additionally, integrated within the phases are elements of narrative analysis (Kim, 2016; Polkinghorne, 1995) Braun & Clarke (2022) described the six phases of reflexive thematic analysis as follows:

Phase 1: Familiarizing yourself with the data set

Phase 2: Coding

Phase 3: Generating Initial Themes

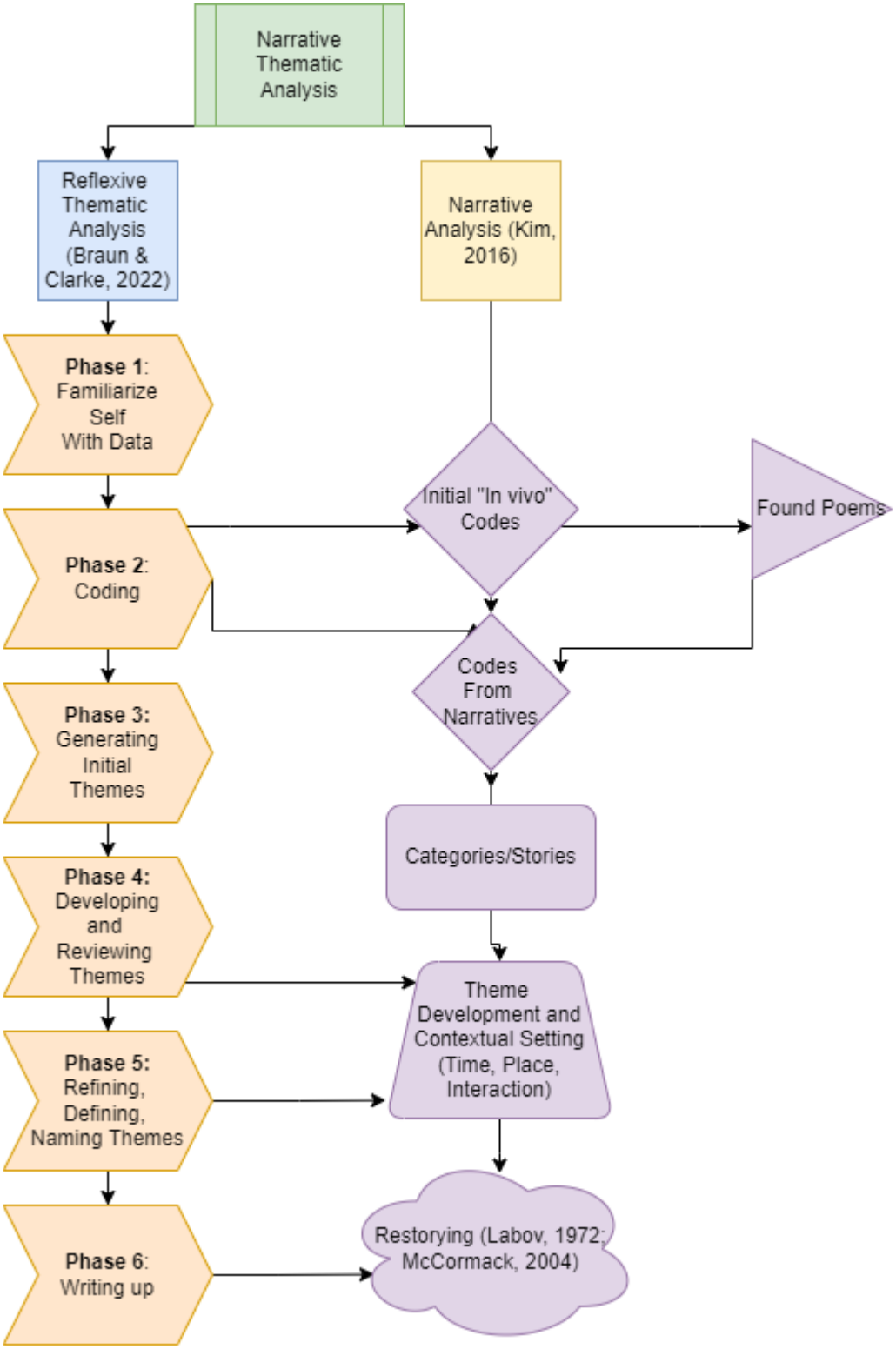
Phase 4: Developing and Reviewing Themes

Phase 5: Refining, Defining, and Naming Themes

Phase 6: Writing up (p. 35-36)

Figure 3.3

Narrative Thematic Data Analysis Process



Interview and focus group data were manually transcribed. The transcription tool, HyperTRANSCRIBE, was used to assist with audio pause and playback to facilitate typing focus group and individual interview transcriptions. Each interview and focus group transcript was organized in a separate Word document with a column to note codes and a column to note impressions and thoughts. Data were analyzed through an inductive process (Cohen et al., 2018). In vivo codes were identified during the first round of coding within each participant's transcribed document (Saldana, 2021) (see Table 3.3), while noting recurring words or concepts and possible categories. The first round of coding was used to stay grounded in participants' words. A found poem using "in vivo" codes (Saldana, 2021) was created for each participant as a way to begin to construct each participant's unique narrative. These poems are included in Chapter Four.

The second round of coding categorized participants' narratives (see Table 3). Interview transcripts were prepared and grouped by stanza made up of like topics. Stories of participants were identified by re-reviewing transcripts and creating codes that represented the stories participants shared during their interviews and focus groups. These stories were not always neatly told in a linear fashion and in the analysis, stories were re-storied (McCormack, 2004).

The researcher analyzed both written notes and digital recordings of data to consider important aspects of the focus group discussions and interviews. Krueger (1994) described a process in which participants verify the information the researcher has gathered to ensure accuracy, and this participant checking and involvement is also in alignment with a narrative inquiry approach (Connelly & Clandinin, 1990).

Therefore, participants were asked to provide input and feedback during the analysis process to ensure the accuracy of data and validity of the interpretation of participants' stories. Participants were provided with a copy of their interview and focus group transcripts for review and were offered an opportunity to clarify or add additional comments or information. Additionally, participants were provided a copy of their found poem, written narrative, and a description of overall themes identified as a result of the research. Participants had another opportunity to review, comment, and provide feedback.

Table 3.3

Sample Codes and Categories

First Round Coding: Sample "in vivo" codes	Second Round Coding: Sample Categories
• "didn't feel invited to the math world" • "I'm not a math person" • "stuff I internalized" • "the correct answer" • "at the kitchen table crying" • "pretty good at math" • "math was always fun" • "being taught at" • "I'm supposed to be good at this" • "self-esteem downer" • "bookwork" • "own my mathematics" • "anxious perfectionist" • "labels on the courses" • "taught old school" • "stigma attached" • "light bulbs went off" • "dad homework" • "going to extra help" • "taught from a book" • "math brain" • "memorize facts" • "practice shift" • "insane first year" • "really like the teacher" • "never looked at a math manual" • "grades on display"	• Parental support • Explore, play, hands-on, student directed • Societal views of math • Textbook curriculum • Achievement and Ability • Influence of Teachers • Challenges as a learner of math • Moving on/lost • Relevance/applicability • Right/wrong answer • Impact of Covid • Confidence/motivation • Aha moment • Up for the challenge • Access and inclusion to math ("club") • Future focused with students • Errors/mistakes • Expectations • "Fun" vs. "drudgery" • Failure • Practice disconnect • Jump in, get feet wet • Math reasoning

Limitations

As data collection commenced a significant challenge was observed in participant recruitment. Due to the initial low number of interested participants, the study was opened to more candidates. The original proposal had anticipated being able to hold multiple focus groups of pre-service teachers; however, this was not feasible. These limitations will be further discussed in Chapter Four.

A central focus of narrative inquiry is on the experiences of individuals which cannot be readily generalized to broader populations (Marshall & Rossman, 2011). Results of this study provide insights into how individual pre-service and novice in-service elementary teachers perceive experiences and develop their capacities to learn and teach mathematics, and commonalities between their experiences emerged. However, different populations or even different individuals within the same population may have divergent experiences. The purpose of this research was exploratory and therefore a qualitative design that enabled the researcher to understand individual perspectives was appropriate. Follow-up studies with different populations or other approaches with a larger sample of participants may be more appropriate to further examine experiences.

Another important factor to consider is the issue of researcher bias throughout the research study. As noted in a previous section, I reflected on my belief system as a researcher. I also gave considerable thought to my own “stories” and experiences I brought to the research and how this could influence my role as a researcher. Throughout the research, I made a continued effort to approach this research with openness and curiosity. To this extent, I remained conscious of my own preconceived

ideas and past experiences with the subject. When I drafted and asked questions, I considered if any biased opinions were embedded within what I asked or did not ask participants. During interviews as I kept field notes, I separated observations from impressions and reflected on why I concluded a certain impression. As a narrative inquirer studying others' experiences, I was thoughtful of how I was experiencing the participants' stories. As Clandinin and Connelly (2000) suggested, I kept a column in my field texts for reflections on how I was "experiencing the experience of being in the inquiry" (p. 88).

Summary

This qualitative research study utilized a narrative inquiry approach to examine how pre-service and in-service elementary school teachers perceived their own development as learners and teachers of mathematics. The purpose of this study was to understand how emerging elementary educators experienced mathematics learning and teaching, and what experiences and conditions they viewed as purposeful to their own growth. An outcome of this research is to present additional insights into these experiences and how elementary school teachers make sense of these experiences. These insights can then be used to inform teacher preparation programs to take meaningful action to improve practice. Furthermore, the research suggests directions for further studies into mathematics learning experiences for pre-service and new in-service elementary school teachers.

Chapter Four: Description of Findings and Recommended Actions

This research examined how elementary pre-service and new in-service teachers perceived their development of mathematics learning and teaching as they became elementary educators. This study explored the experiences, relationships, and conditions that pre-service teachers and new in-service teachers viewed as influential in developing their capacities to learn and teach elementary mathematics.

This qualitative study utilized a narrative inquiry approach to explore the perspectives of participants and to answer the research question: How do pre-service elementary teachers and in-service teachers perceive the experiences and conditions that influence their development as learners and teachers of mathematics? Narratives were gathered through individual interviews with each of the nine participants and through two focus groups, one group of pre-service elementary teachers and one group of early career in-service teachers. As described in Chapter Three, a narrative thematic approach (Butina, 2015) to analysis was taken to understand and analyze the data. In vivo codes (Saldana, 2021) were used during the first round of coding and a found poem was created for each participant to remain grounded in participants' words and represent a summary of their individual narratives.

The second round of coding categorized participants' narratives. Interview transcripts were prepared and grouped by stanza made up of like topics. Stories of participants were identified by re-reviewing transcripts and creating codes that represented the stories participants shared during their interviews and focus groups. The narratives of participants were then re-storied (McCormack, 2004) to present an individual's narrative chronologically. The following section introduces each

participant initially by sharing their individual found poem, followed by a brief written narrative. Subsequently, this section will describe major findings and implications through a discussion of the research themes.

Participants

Sarah

My Mt. Everest

Math has always been a real struggle
for me;
Math seemed like drudgery

I didn't feel invited to the math world
Stuff that I internalized;
I'm not a math person;
Is just societally accepted rhetoric
I'm not a math person
That's just how we talk about math

I just didn't understand
I couldn't read it
My logical reasoning is not fluid

My aha moment
I realized I could do it
Engaging professor made the abstract
concrete;
It felt like unlocking tiny little doors
It was all coming together

Math became solving puzzles
I *Became a Math Person;*
I loved math

I'm going to climb that mountain
I'm going to get it
Like walking through the fire
to get to the other side
I had to fight to get here
so can you
Feel like a mathematician
Become a Math Person

Everybody is invited to the club
Anyone can become good
You can build those muscles
You can too
Everybody is rising up together
With the same toolbox
Be a mathematician with me.

Sarah had a love for school and learning from a young age. She described,

> I started school you, you know, as usual at like six, seven…six, and I, like instantly knew I wanted to be a teacher. I fell in love with my teachers. I thought school was the greatest thing. I had a happy house at home, but I just found more happiness at school. It was a really fun place and I kept that in my heart for a long time.

She recalled playing with mathematics tools in early elementary school and could not wait to get to school to play with them.

Sarah "struggled with math" when she was a K-12 student. She described:

> I think, Somehow, my logical reasoning and my way of finding patterns, my number literacy, it's not, uh, it's not fluid when I try to read…when I was younger even coming into math, just, like, I couldn't read it. It was really hard. I wanted to know why and for some reason, like, I find language arts to be more internalized in the way I communicate. I don't communicate a lot of mathematical thinking, so I'm not as practiced in it.

Sarah identified middle school as the time she remembers mathematics becoming challenging for her. She recalled,

> I think algebraic thinking is when it got hard for me. It was just timed with a lot of socio-emotional learning of pre-puberty/puberty. I was distracted in a big way. I didn't feel great about math anymore. I wanted to talk about other stuff. It became really hard to conceptualize what the point of an exponent was or why it was able to do that, and I was very frustrated by the idea that the only way, and still the only way, I know how to find a random square root is with a calculator…It was like I didn't feel welcome to the club anymore because I didn't understand.

At the time, she internalized these feelings of not being invited to the mathematics world because she believed she was not "good at math." Mathematics as an adult was still challenging, but she had come to enjoy the challenge.

Sarah was graduating from her teacher preparation program with a four-year bachelor's degree in elementary education. She was a non-traditional student who first attended college after high school. She reflected that this was not the right time for her to pursue college. After putting higher education on pause, she worked in restaurants for many years. Once she got married and began to establish her own family, she started to "remember her dream" of becoming a teacher. The time was right; she felt supported and had a renewed sense of purpose.

Being a non-traditional student presented its own set of unique challenges and considerations for Sarah. Sarah approached higher education with intention. She felt the need to "be pragmatic with her decisions" and recognized that if she were to spend money towards a degree, she wanted it to be in something she found challenging. Money would be well spent if it went towards learning something new and taking advantage of faculty expertise in areas that were unknown to Sarah.

Sarah described her "aha moment," when she realized she could do mathematics and that she enjoyed it. She took a mathematics course at a local community college with an engaging professor she describes as "an incredible mathematician." Sarah fondly remembered,

> He was teaching us things like how to quantify different levels of infinity and making that accessible. And things like how to know any angle a light might shine in a room regardless of the shape of the room. Things that were mind-blowing to me that seemed like made sense, you could figure it out, but I had no idea how you would get there. He made that…it sparked my interest and from there I just, like, decided this is hard, but I can do it.

With Sarah's new sense of confidence and enjoyment of mathematics, she continued to take courses and embrace the challenges.

As she planned to enter her own classroom after graduating, she was driven to help support students to see themselves as mathematicians. Sarah came from a place of understanding failure and feeling not welcomed to the mathematics world, but that inspired her to support her future students.

Chloe

We all Need Math

I was always pretty good at math
Math is a relative strength
I was taught math was important;
Was something I tried to be good at

The way I was taught
You gotta know it
Do it this way
Get the *correct* answer

My biggest pet peeve
Is a worksheet full of word problems
Writing numbers on a piece of paper
Get paper back with Xs
Great I suck at math
Just makes me annoyed
That's not helpful
It doesn't help fix mistake

Bringing home math homework
Fall apart
If it was harder one day
At the kitchen table;
crying
With my parents
I didn't know how to do it
They didn't know how to help
Wasn't how they learned

Shifting that role from student to math teacher
Putting ourself in the seat of the student

Woah, what did you just say?
What in the world are they doing?
I don't know if I can help
A lot has changed
The way they are teaching now
Not having kids be in time crunch situations
Able to have conversations
Math reasoning can open up conversations
There is not one way to solve
About how you got there;
Why solved it this way
Push them a little deeper

A really useful class
How do you teach a kid to count?
My mind was blown
Kind of questions I never pondered
A new way of looking at math
I really had to rework things I learned
To understand to teach it
Making self familiar first

We all need math--

Baking at home,
At the store;
You have to know how much.

It's not about whose right or wrong
Let's play and explore
We all need math

Chloe was a graduating senior in an elementary education program. During the last semester of her program, she student-taught in a first-grade classroom. Chloe initially saw herself teaching older grades, such as fourth or fifth grade, but after subbing in the younger grades, she found she enjoyed this age quite a bit. She had accepted a job for the following school year in a third-grade class close to her home.

Chloe described that she was "always pretty good at math as a kid," but she had some challenges. She remembered bringing home her mathematics homework and sitting around the kitchen table with her parents, crying. The frustration increased when Chloe tried to approach mathematics from the strategies being presented to her in class, while her parents were approaching mathematics the way they had learned. She described,

> I definitely remember those times where bringing home a book and reading it to my parents was fine, bringing home spelling words to practice—that was always fine, but I think that math is definitely one of the ones…if I had to bring home math homework, like, I remember a lot of nights being at that kitchen table with my parents crying about my math homework because I didn't know how to do it and they didn't know how to do it because the way it was being taught wasn't how they learned.

Chloe spoke of the parental support she received beyond assistance with her homework. She felt that her parents valued education and mathematics when she was growing up. She recalled applying mathematics in her everyday life. When her family

was at a store, her parents would ask her to figure out how much money was needed to pay for what they were buying. She also saw mathematics in the kitchen when she was cooking at home while measuring ingredients.

Chloe spoke highly of her mathematics teaching preparedness in her teacher preparation program. She recalled having to ponder questions she had never thought of before, such as how to teach a student to count. She discussed one of her early methods courses,

> I remember, like, the first day my mind was blown. We were sitting there and he put a number of objects up on the board and he was like, "how many are here?" and I was like, "that's easy—5" And he was like, "oh, well how do you know?" And I was like, "I just counted them. That's really not that hard. And he was like, "but if a kid had never learned to count, then how do you teach a kid to count?"

She discussed what it was like to relearn many of the concepts she learned from her childhood and learn to use and teach new strategies:

> The way I was taught to do math was very different from the way they are teaching it now. So, I really had to rework the things that I had learned in order to understand how to teach it to kids.

Chloe emphasized the importance of relevance and applicability when learning and teaching mathematics and the importance of "getting kids to be more hands-on and apply the math to real-world, practical scenarios."

Heather

All about the Math

Earliest memories Math was always fun

Love doing lattice all over the place
Math is fun to do

Some people said don't like math
Very black and white
It's boring
Not fun
Don't understand

Didn't love geometry or pre-calc
Kind of hard
It was the pre-calc teacher
Just do PowerPoints
Packets of just words
Having to have formulas memorized
Wasn't practicing math
Being Taught at
Not actual doing or learning
Being talked at

Big self-esteem downer
It kind of hurt
I'm supposed to be good at this
Should make sense to me
Always been good at math
I don't understand why
Why doesn't it make sense to me

You guys already know this stuff
You've been doing math for so long
Kept moving on
Starts getting harder
Not really understanding what learning
We just keep going
Like a cycle

The way math is presented
Bookwork
Same thing every day
Do these pages
On your own
Exact same thing
repetitive
Even boring for me

Bookwork with those programs
How it was for me

Issue between me and my dad
Do math homework with him
Dad is math person
Get into big argument
Trying to show him lattice
Not how supposed to do it
Not how we are learning
I can't help you
I don't understand
That doesn't make sense
Was a struggle
Weren't able to do together
Ended up doing it alone

Loved that math teacher
He was all about math
He wanted you to love it
Learn it
Understand it
You could love it
A great teacher

Never thought of exploring math
before learning it
Surprised to learn
Give students manipulatives
See what they do with it
Seeing different ways
Show other students how to do it
Finding different ways to show them

Knowledge increased about math
Reteach myself
Didn't remember being taught to add
Weird to think about
Muscle memory thing
Reteach self
Push into teaching
One of my favorite subjects to teach

Heather was a pre-service elementary teacher who was about to graduate from her teacher preparation program. She would graduate with a four-year bachelor's degree and had applied to graduate school for a Master's in Special Education at the same institution. Heather fondly remembered learning mathematics. It was something she always enjoyed and felt successful in her capabilities as a mathematics learner. She described mathematics as "fun" at multiple points during our interview.

> My earliest memories were probably like third, fourth grade. I've always really enjoyed math. Math was always fun for me. But third or fourth grade is when I distinctly remember. We were learning multiplication and division and all that. And when I was in third, fourth grade lattice was the big thing being taught. So, we used to love doing lattice problems all over the place, like all over the classroom…we would do them for fun.

Heather described much of her experience as a student consisting of learning from a mathematics program. She referenced "bookwork" as a prominent component of her mathematics education and saw similarities in the school she completed her student teaching. Students often engaged in teacher-directed lessons and engaged in independent practice in their workbooks.

Although Heather excelled in mathematics for most of her academic career, she faced challenges in high school. She recalled not enjoying proof writing or her pre-calculus class. She recalled her pre-calculus teacher presenting mathematics in ineffective ways:

> We just had to memorize everything for the most part, which was a lot for pre-calc. He would also just do PowerPoints and just present us with this packet of

> just, like, words. Not even actual math. So, it wasn't even like we were practicing math in the classroom. Just being taught at.

When Heather faced challenges in Geometry, it impacted her self-esteem. Since she had viewed herself as being a highly capable mathematics student, suddenly not understanding or performing as well as she had previously done was confusing and disheartening. She described, "It kind of hurt because I was, like, I was supposed to be good at this. Like, I've been good at this for so long." She sought help from her peers and was relieved to grasp concepts in mathematics courses that followed Geometry. Heather believed there are certain areas of mathematics that she was better at and better suited for, while other areas of mathematics, such as geometry and pre-calculus, were just not for her.

In Heather's teacher preparation program, she enjoyed student teaching and described mathematics as being one of her favorite subjects to teach. Heather reflected on how she learned mathematics, which she described as very "black and white." She did not recall approaching mathematics using different methods or exploring mathematics. She recounted learning about encouraging students to explore mathematics and engage in interactive ways:

> I think I was surprised to learn about just giving students this thing, like a manipulative of something and not telling them what they were actually learning about yet and just seeing what they'll do with it. In classes, we learned how sometimes they'll just know or they'll do stuff without actually realizing they are actually doing it. I just feel like I never thought of exploring math before actually learning it.

This type of learning was different from how she learned mathematics. Her knowledge of how to present mathematics to students increased. However, she wished her mathematics coursework went deeper into the mathematics concepts. She felt the emphasis of her mathematics coursework was on planning and writing mathematics lessons and how to present the mathematics to the student.

> Instead of just learning how to write a lesson plan, the unit, or writing what we would do, it would be better to learn more about the subject. Like what the subject actually is and things in it. Because we weren't in elementary school in a long time.

Heather wished there was additional preparation on revisiting mathematics content needed to effectively teach her future students.

Meghan

Owning my Mathematics Skills

Math
I have a funny relationship with it
My mom would always say
Math is your sister's thing
You do the writing

My elementary years
A lot of skill and drill
Never did 10 frames
Don't remember arrays
Remember memorization

Anxious perfectionist
Afraid of not know the answer
Afraid of asking for help
That would be me being dumb

Honors, average, basic
Were labels on the courses
Pre-calc teacher was fantastic
I felt our teaching was reaching out to us

Life has brought me to different paths
Interned in D.C.
Goal was non-elective politics
Realized all the stereotypes were true
Had bills to pay
Receptionist at a company
Became web designer/developer
Stay at home mom

Daughter in 1st grade
Starting subbing in local district
Friend an advisor,
Said "going to get you an education degree"

Fantastic professor
"Always ask about the why"

Told me I would be an excellent math teacher
Wait, me?
That outside perspective
I could then internalize
Own my mathematics
I do this well

5th grade, 1st grade, 3rd and 4th
Every grade—this is the one I love
Asking them "the why"
Feel comfortable being wrong,
Learning from mistakes

A lot of students grumble
"Math is too hard"
But also some success stories
"I can do math"
Some will still struggle

When hear math coming
Most of them embrace it at some point
So that was a triumph

Don't always agree with how the program tries to teach
Use application in real life vs. what the textbook says
Focus on problem solving
Embracing different methods
Getting into that why

First time this fall
Get to reteach a grade
Okay, I remember
I did this before
Owning my mathematics skills
My confidence will be even higher this year.

Meghan had completed her second year as a classroom teacher. Meghan described teaching as a mid-life career change. Meghan's undergraduate degree was in German with a minor in Political Science. She initially saw herself going into non-elective politics. Upon graduating college, she interned in Washington D.C., and quickly realized she no longer wanted to work in the political environment. However, Meghan needed to support herself financially and accepted a job as a receptionist. She made connections within the company and ultimately became a web designer/developer. Meghan maintained this job until she decided to stay at home with her children when they were born.

When Meghan's daughter was in first grade, she decided she would return to work outside the home. She began working as a substitute teacher and continued subbing for seven years, working in pre-kindergarten through eighth-grade classrooms. While she was substitute teaching, she decided that teaching was

something she wanted to pursue. Meghan had a friend who was an academic advisor who urged her to obtain a master's degree in education. Meghan initially thought she wanted to teach middle school social studies, yet through her student teaching experiences, she discovered she enjoyed the elementary age range. She completed her degree gradually, taking one course at a time as she continued working as a substitute teacher and caring for her family.

A long-term substitute position became available in February of 2019. She was unsure of the position initially because it was a first-grade position; however, she found that she enjoyed this age group. Meghan was with this group of students for a few short weeks before her school shut down due to the Covid-19 pandemic. The following year she found a role as a third-grade teacher at the district she currently worked in. She was able to loop with some of her class the following year as a fourth-grade teacher and described she would return to third grade the following year.

Meghan emphasized understanding "the why" in her mathematics teaching and this was something meaningful she took away from her pre-service mathematics coursework. She wanted her students to utilize reasoning and describe their thinking. Meghan did not always see herself as a capable mathematics student. She described when growing up her mother would encourage her to pursue writing and emphasized that mathematics was her "sister's thing." Meghan also described being anxious about mathematics during her K-12 experience, being concerned that she would be perceived as unintelligent. Meghan discussed that it has taken time for her to "own her math" capabilities and see herself as a mathematician. As she was heading into her

third year teaching in her own classroom, she was feeling more confident and was recognizing her capacity to teach mathematics.

Tara

Taught Old School

Local private school
Huge influx with covid
In person the whole time
6 feet guidelines
Couldn't mix adults
Couldn't leave your kids
You and your kids

Outside a lot
Had a wagon
Haul stuff outside
Kids in camp chairs with clipboards
Don't know how learned anything

God, it was awful
It was rough
Stay away from each other
Don't share
Things you never want to teach a kindergartener

I was taught old school
No reason why you carry the 1
This is how you do it
Learned one way

I'm that odd person

I really do enjoy math
Has that stigma attached
Either you like math or you don't

Tons of conversations with parents
Don't like Common Core
"New math"
Nothing new about it
New to them
Shut down
They are afraid
I.Don't.Know.

Interesting learning different strategies
Never taught that way
There is no good at math, bad at math
30 years later
Math class for me
Never seen this before in my life
Didn't learn that way
Enlightening to me
Inquiry-based
Bringing in everyday life in math
Light bulbs went off
Felt more secure, more confident
Willing to try and share.

Tara was a non-traditional student who returned to college in her 30s. She completed her bachelor's degree in elementary education several years ago. After completing her degree, she took a year off to care for her newborn child. She then started teaching pre-kindergarten at a local private school that her son had attended. The following year she taught kindergarten and this past year she taught second grade.

Tara recalled always enjoying mathematics growing up and described being taught mathematics differently from how students are taught today. She described,

> I enjoyed math, so it wasn't something that I…I know some people tend to dislike it, but I always enjoyed it. But I was taught old school and no reasons why you carry the one. This is how you do it, right? Teach you how to do it, this is how you do it and you have to get the right answer. You know? And that is how I learned math and you didn't learn different strategies then.

Tara found it interesting to learn different strategies in her mathematics methods coursework and recalled it being "enlightening to me because I didn't learn that way." She described, "I was like that totally makes sense! I never realized the whys behind it, you know, or any of that." Her understanding of mathematics reasoning and place value grew. Tara wished she had had an opportunity to have more classroom experience during her teacher preparation program and had gained experience with understanding and utilizing a mathematics program. She described, "I think it would have been helpful. Kind of more experience with [a program] and even using it in the classroom somehow, in the college classroom, you know? How to supplement."

Tara recognized there is a stigma surrounding mathematics and discussed conversations she had had with parents about Common Core State Standards for Mathematics. Many parents Tara had engaged with were uncomfortable or confused by what they considered to be "new math." Tara said she tried to explain that the mathematics itself is not new, but the strategies and the way mathematics was being presented is different. The conversations she had with parents seemed to be productive and she was able to provide an explanation that was well-received.

Teaching during the Covid-19 pandemic was a challenge that Tara discussed. The private school that Tara worked at remained open for in-person learning during the pandemic. Tara described,

> God, it was just awful. I wish those poor kids never had to do that year. It was just rough. I spent the whole year yelling at kindergarteners to stay away from each other and don't share. You know, all the things you never want to teach kindergarteners.

She recalled being isolated in her classroom with her students, unable to interact with or welcome other adults into her classroom. She conducted lessons outdoors under a tree with kindergarteners in camp chairs with clipboards. Tara would haul her wagon with a portable whiteboard outside and each child had a box of their own manipulatives to use. Looking back on that school year, Tara was in awe that students were able to learn.

> Honestly looking back, I don't even know how they learned anything. Like insanity now. This year just feels kind of like a cakewalk with second graders…we are still outside a lot, but they are not five in a camp chair.

Covid-19 created numerous defining memories for Tara during her first years teaching.

Naomi

Time for the Dad Homework

I'm a person who likes to find an answer,
Math is my favorite subject
Math is fun;
Always liked it.
A lot of kids hate it;
I never felt that way
My dad would give me addition problems
"It's time for the Dad Homework";
Standard algorithm
Carry the 1
Not a fan of other methods

"It's confusing you"
"Do it this way"

When don't understand,
And the teacher moves on
Get frustrated,
Just kind of lost,
Easy to get lost

7th-grade teacher--
She never explained
Whizzed through so fast
I was lost all the time.
Struggling in math
Going to extra help
Hardest school had been
Trying hard

I'm slow when it comes to learning
Last person to finish a test;
I'm that person.
Test out of grade-level math
I was one grade level above--
Accelerated pace
Too fast

My experience with math;
Very workbook-based
Taught from a book
Time for math
The only way I remember

Math methods course--
Show us student work,
Figure out what they did wrong
Thought that was fun,
was really useful
Can learn a lot from common errors

Want it to be fun;
Not the dreaded subject,
Want it to be collaborative
Ask a friend to help you learn,
Don't have to be struggling by yourself
Make it more engaging
Isn't just doing problems on a paper
Take math and apply it
Instilling high expectations

Naomi was a pre-service teacher who completed all her coursework in her teacher preparation program and only had her full year of student teaching to complete in her master's program. Naomi's undergraduate degree was in Human Development and Family Studies. Naomi described mathematics as her favorite subject and she recalled, "I just think math is fun. I liked doing algebra problems and stuff like that…I know a lot of kids hated math and that it was the worst subject, but I never felt that way. I liked it."

Naomi recalled her father took an active role in supporting her with her mathematics growing up. Naomi's father would help her with homework and give her supplemental mathematics work to complete during school breaks. Naomi described,

> I always remember my dad would give me addition problems because we would have summer and he would be like, "Okay, it's time for the Dad Homework." And he would give us addition and subtraction where we had to, like, carry the one and take away.

Although Naomi had a positive mathematics experience overall, she recalled challenges when mathematics felt rushed. She described needing time to process and think through her work and have time to practice to master concepts. She was placed in an advanced-level mathematics course beginning in middle school and felt like the class was taught at an accelerated pace with the assumption that the students placed in the class would be able to learn faster. Naomi described,

> I think I'm kind of slow when it comes to learning. Like, I'm always the last person to finish a test. I'm *that* person. I think it was just too fast. Part of it was after sixth grade you could test out of grade level math and be one above. So, I was one above, so I skipped one and went to pre-algebra. So, I think they just expected if you were one-above that they could go really fast.

Overall, Naomi did not find her mathematics coursework in her teacher preparation program to be very meaningful. However, she did find it useful to learn from student errors. She contrasted her mathematics method courses to her literacy method courses and wished the mathematics courses mirrored the structure of the literacy courses and approached mathematics learning from a developmental perspective. Naomi suggested,

> It would have been nice if we went from emergent mathematics skills to more advanced. Like if we took a more developmental track that would have been

> helpful, but we didn't do that…yeah everything that goes on a developmental track makes so much sense to me and when things are all over the place, I'm like, what age is this?

Naomi was looking forward to her student teaching experience in third grade and learning from her cooperating teacher. When visiting the classroom, she observed her cooperating teacher engaging students in an interactive mathematics lesson that got students moving to music. Naomi hoped that mathematics learning for her students would be fun and something they enjoy. She looked to the future by describing:

> I want it to be fun and not something that you do in your workbook and not the dreaded subject. I want it to be collaborative because I feel like there is a lot of value in that and you don't have to be in your textbook struggling by yourself.

Amelia

It's more about the process

Black sheep of the family--
Math is prevalent;
Parents are engineers;
Sister graduated with math and
computer science degree;
Other sister has a degree in accounting.
I didn't have quite the math brain;

I hated trying to memorize facts
Feeling like I wasn't very good,
Didn't understand
Having a really tough time
It didn't come naturally
Was always more of a challenge

Sitting in a meeting
with my mom and math teacher
A conference that they called
"You can do math!"
My teacher, he believes in me
Going to the bathroom and crying
Felt I could do math after that

My parents care about education--
Dad asked for the teacher manual,
Helping me make study guides,
Talking to my teachers.
Vivid thing--
Asking for help on homework;
Mom waking up in the middle of the night,
Couldn't figure it out,
Called my grandfather,
Figured out in middle of the night,
Always had to solve the problem,
Make study guides
Had to help out
Were in a good place

Heavily involved in college
Really busy,
RA, theater,

Very little sleep,
Whirlwind.

Always known I wanted to be a
teacher,
Wow, that's been a moment
Realized misconceptions
Set me up,
Help me think about what is algebra?
What does it mean?
Why we do certain things

Graduated;
Didn't feel ready,
Thought of actually being a teacher
Became overwhelming

Insane first year
Everyday Math throwback
Struggled teaching math
So confused about the goal

Last year I struggled
But how does that fit in with what I
have
As instruction and what I have to teach
One experience vs. what I could do
consistently

Professional Development
A big mind shift for me;
It's not a worksheet,
Show us techniques,
Little things,
Practice shift,
"Come to the perimeter of the rug"
Never thought of before

You need to struggle to learn;
Be by yourself,
Try it out to see
Experiencing it and failing a little bit
and trying it again

I don't totally get it yet
I've grown a lot,
Become more confident
You don't have to have the answer
It's more about the process.

Amelia was a first-grade teacher who had completed two years of classroom teaching. After she completed her undergraduate degree in early childhood education, she did not feel ready to teach and enrolled in a master's program in special education. After completing her master's program, she accepted a long-term substitute case manager position at the school she interned in. The following year, Amelia started as a classroom teacher. In her undergraduate program, Amelia gained quite a bit of classroom experience through her pre-practicum experiences. However, she described reaching the end of the program and feeling overwhelmed with the thought of "actually becoming a teacher."

> I have always known I wanted to be a teacher but when I was there, I was thinking, and I didn't feel ready after student teaching to get a job right off the bat…I was very overwhelmed when I was student teaching and had very little sleep. I remember being observed after having like four hours of sleep and was like, that probably wasn't good. So, it just felt like a whirlwind and so fast that even though I had a lot of experience in a classroom, like, we had to do a lot of pre-practicums in our program and they really pushed us to take over a lot more…It was really overwhelming with, like, we had to write a lesson plan for every single thing we taught when we were in takeover…So it just felt like I was doing so much paperwork and trying to figure out how to be a teacher that when I thought of actually being a teacher, it became overwhelming.

Amelia described herself as not having the same inclination towards mathematics as the rest of her family members. She described that both her parents were electrical engineers and she had a sister with degrees in mathematics and computer science and another sister with a degree in accounting. "I always felt like the black sheep of the family in terms of math. I didn't have quite the math brain that they did." She found mathematics to be challenging in her K-12 experiences. Memorizing facts, in particular, was difficult for Amelia. Amelia's parents cared about her education and made efforts to help and support her by helping her.

> I mean, my parents care a lot about education, so that's just… I remember them always helping me make study guides or talking to my teachers when I wasn't doing well. I felt like math was the subject they would reach out and help, get me help in.

Teaching mathematics during her first year as a classroom teacher was challenging, partly due to the Covid-19 pandemic, but also because she was learning to understand the mathematics program her school was using. Amelia found it difficult to understand the focus of the program and leaned on her colleagues to help answer questions and provide guidance. The past year, Amelia's district adopted a new mathematics program and also partnered with a mathematics consultant. Amelia spoke very highly of the mathematics consultant who provided her with tools and strategies she could readily apply in her classroom. She spoke of a disconnect between her teacher preparation program and integrating what she knew to be best practice consistently once in the classroom. However, the consultant was able to provide tools and ideas she could clearly implement.

> I just love [the mathematics consultant]! Just even the little things. She'll be like, "come to the perimeter of the rug." And, like, I've *never* thought of that before! Instead of just sit in a circle on the rug, you are teaching perimeter so when you get to perimeter later on, "oh yeah, I know what I'm talking about, that's the outside edge—that's from first grade! My teachers said that all the time" Or even just attention getters. She'll put ten on her head with, like, cubes. "I have five on this hand, how many on this hand?" Just quick things you can do that are simple, but they are what you can do in the classroom every day.

Amelia described gaining confidence in her ability to have mathematics discussions with her students. She discussed the benefits of needing to struggle to learn. She used to view failure as something negative but described now recognizing it

as an opportunity to grow. She explained she learned a great deal through practice, "failing a little bit and trying it again."

Jon

Bing, Boom, Bang: It's all Good

One of those things you say--
"I hate math"
Don't think I ever hated math,
I guess I could say indifferent.
Academically I did pretty well,
My math skills are average

In middle school,
I started to struggle in math;
At the kitchen table with my dad,
Trying to go over homework together,
Getting frustrated,
To the point of tears
He wasn't doing it the right way
Super frustrated
Close the book
Let your teachers know you couldn't finish

Middle school was tough,
Math was confusing;
Easier for other students around me
I was noticing that.
Everyone knew what to do,
I had no idea
I was the only one struggling
Felt like that
Which I'm sure wasn't the case

There's a formula--
If you stick to the formula,
Plug numbers the right way,
It kind of flows.
You get through it
Bing, Boom, Bang
Got the answer,
It's all good
I look at it like a puzzle,
All the pieces fit together

Junior year, I really liked the teacher
She made an effort
Took an interest
Cared you did well
Made an effort to connect
That drove me more
I wanted to do well

Some teachers didn't make an effort,
You weren't seen as much;
In the background
Got to get through stuff
Can't get to them all
Wasn't much connection

Covid semester--
Second guessing it in my placement
Professor wrote to the Dean
Academic watch
Didn't feel I was taking it serious enough
I was struggling a lot

When Covid happened
I needed more structure
Online learning was difficult for me
Kick you out of the program

Online learning was hard
Zoom,
Sending you stuff to read
Teach yourself,
Time in the program on the ropes

It all worked out,
Turned everything in on time,
Last semester sealed the deal.
Really great teacher worked with;

Threw me into it
Always right there to guide

Wish there was more of
What it's like to be a teacher,
What it looks like in the classroom.
Get thrown in,
Gotta figure it out;
That's how I work best

Never looked at a math manual,
I wish that would have happened;
Dive into the curriculum
With that guidance from a seasoned teacher
Dive into that book
Wish I had it sooner

My first real year of teaching
I'm learning new curriculum
They're learning new skills
Math has been a struggle
Hybrid, In person, 6 feet apart
Bing, Boom, Bang

Jon was entering his second year as a third-grade teacher. He graduated from his teacher preparation program and obtained a job as a long-term substitute at the school where he was teaching during the time of the interview. Jon described doing fairly well all around academically when he was a student; however, he did encounter some challenges in mathematics. Jon recalled mathematics beginning to become more difficult in middle school. He remembered doing his homework at the kitchen table with his father:

> I remember in middle school—I think it was seventh grade, and I remember I'd be at the kitchen table and we'd be trying to go over homework together and he'd be getting frustrated because I was getting frustrated because he wasn't doing it the right way or whatever. So, I remember that being really frustrating. Like, to the point of tears.

At school, he started noticing other students in his class appeared to grasp mathematics concepts more readily and this was something that Jon internalized at the time.

Jon recalled one teacher from high school who made an effort to get to know him. He described that she cared that her students did well, and this motivated Jon to want to do well in the class as well. He recalled this teacher coming to his basketball games and building a positive relationship with her students.

> I mean she just took an interest in everything and everything every student did. Like, I played on the basketball team and she'd always ask me about the game last night. She always went to the game so she'd always talk to me about the games and would ask me what I thought about this play or that play or whatever. And I remember one game in particular. It was a really close game with a really good team and they went into overtime It was a home game and the atmosphere was really cool. And I was exhausted afterwards. It was a super long game. I was just spent and just sore and I came in and math was first thing in the day, in the morning. And she just looked at me and said, "oh you must be exhausted. You look exhausted. That was a good game. It was a fun game to watch." And I was like, "yeah, yeah, it was." And we just kind of went on with the day.

As a teacher now, Jon reflected that he appreciates how much this connection means even more.

Covid-19 presented challenges for Jon during his teacher preparation program. Online learning was difficult, and he was second-guessing his teaching career at one point. However, during his last semester of student teaching, he had a great experience and connection with his mentor teacher. Jon described,

> I went in and it kind of sealed the deal for me. It was like okay, yeah, this is what I want to do. The teacher that I worked with was really great. We got along really well and she just kind of threw me into it and said, "Hey, do you want to teach this today, you want to teach that today?" outside the required lesson I had to teach. So, I got a lot of experience teaching.

He was able to "jump in" to the curriculum and had the opportunity to gain valuable hands-on experience in the classroom. He began teaching during the pandemic, which presented its own set of challenges.

Jon found navigating the curriculum his school used challenging in his first year. He wished he had the opportunity in his teacher preparation program to have greater exposure to a mathematics program. He also described that differentiating the needs of diverse learners in his classroom had been difficult. He had learners with a range of different skills and understanding and meeting the range of needs was difficult. As he prepared to enter his next year of teaching, he was looking forward to the whole school's implementation of a Multi-Tiered System of Support model to help him support his students at various levels of understanding.

Brandi

Grades on Display

Remember thinking fondly of math
It was fun to do
Was a positive experience

A positive relationship--
7th, 8th grade math teacher looped with us
He valued me,
Helped me out,
Let me know he cared;
He met me where I stood

A memory I have
Grades posted outside classroom
Did not like that
Uncomfortable
Going up to list,
Follow my finger,
Grades on display.

One teacher that I didn't get along with
High school football coach

There just for that
Got along with boys
Not so much the girls
Hard to learn when your teacher doesn't like you

Frustrating--
I went for extra help
Was there everyday
See how hard I'm working?
Still didn't make him like me

Junior year started competency-based grading
Allowed to retake assessment as many times as wanted
Could get a 100 on every single thing
I just didn't work as hard

Advanced class--
Started to struggle,
Can't keep up with advanced.
Regular class--
Didn't feel challenged,
Bored,
Checked out.
Can't find a happy medium

Oh my god!
We never learned this way
Not teaching the same way anymore
Don't remember learning place value
Didn't know it was a 10
It was just a 1 to me
It makes more sense than the way we learned

It's hard learning curriculum--
Kind of tells you what to say and do
I have this script,
Can say this script;
Read the lesson,
Implement it
I haven't needed other knowledge.

Leading a discussion is hard
If I don't know what I'm doing
Behaviors throw off the lesson
I start to shut down
It's hard to have confidence

How do I get you there,
To help you understand?
Make it relevant
Applying it to their lives
Not all teacher directed
Fun to do

Brandi was a pre-service teacher graduating with her master's degree. She had spent the last year interning in a second-grade classroom at a pre-kindergarten through eighth-grade school. Brandi described always having a positive attitude toward mathematics growing up. She recalled, "I remember thinking fondly of math. I always thought it was fun and excelled at it. So, I've always had a positive attitude toward math." Brandi was placed on an advanced-level course track beginning in middle school. She had a positive experience in mathematics overall but had some challenges

in high school. Brandi described a negative experience with a teacher who was her school's high school football coach.

> I had one teacher I didn't totally get along with. I think he was the high school football coach and seemed to be there just for that. And like loved the boys and football players. And I started to struggle a little bit with the advanced geometry class. I started to struggle and I had to go in after school and I tried my best.

When Brandi began to struggle with the content, she felt like this teacher did not support or respect her. The next year she was doing better in class, but she still needed to go for extra help. By her junior year, Brandi had moved from the "advanced" class to what she described as the "regular" level class. She was performing better but did not feel challenged in the class.

> It was almost too easy. I could just tell I was, like, one of the only ones who really understood anything that was happening. So, like, okay, I can't find the happy medium, really. Um, you know? I didn't really feel challenged in that class.

Brandi completed her undergraduate degree in Human Development and Family Studies and enrolled in an accelerated master's program in Elementary Education. During high school, she gained teaching and classroom experience through a teaching preparation program at a local regional technical program. Brandi described learning to teach mathematics in a very different way from how she was taught. She described learning about different mathematics strategies as "good exposure," but she had not yet applied much of what she learned in her mathematics methods courses

because the mathematics program her student teaching school used was very structured.

During her student teaching, Brandi described it as challenging to have a high degree of confidence teaching in a way that was different from how she learned. She particularly commented on place value concepts and not necessarily feeling confident answering students' questions.

> Some of the early concepts with, I'm trying to think, place value or anything like that…I mean, I don't ever remember learning place value or anything like that. So, those concepts, super tricky, and I haven't felt super confident in going into it and the kids have questions sometimes and I don't feel confident with my answers.

Brandi had benefited from the guidance of her cooperating teacher, other teachers at her grade level, and the school's mathematics specialist. One practice Brandi enjoyed that her mentor teacher incorporated into mathematics time was a fun mathematics activity on Fridays. Brandi wished she had had more engaging mathematics activities and lessons when she was in school and discussed her hope to bring more active learning activities to her future classroom.

> I hope that I am engaging and as hands-on as possible. I know not every lesson can but…Also applying it to their own lives much as possible just adds to the engagement and overall motivations to do it. And just like explaining how they might use it in the future.

Description of the Findings

Self-Efficacy for Teaching Mathematics Instrument (SETMI) Results

Overall results from the SETMI indicated both pre-service and in-service teacher participants had a strong degree of belief in their capacity to teach elementary mathematics. Results indicated participants had slightly lower levels of mathematics self-efficacy than mathematics teaching self-efficacy (see Tables 4.1, 4.2, 4.4, and 4.5), and depending on the mathematical task, had higher or lower levels of mathematics self-efficacy (see Tables 4.3 and 4.6).

SETMI results seemed to accurately mirror participants' narratives told through interviews and focus groups. The majority of participants believed they were capable mathematics students and teachers. For example, Heather presented as very confident in her ability to do and teach mathematics during her individual interview. This was mirrored in her SETMI result as she indicated "quite a bit" or a "strong degree" of capability teaching mathematics in all her responses. Similarly, she indicated "quite a bit" of capability in doing mathematics in all responses on the survey.

It was surprising to hear the majority of participants speak favorably about mathematics and that they demonstrated relatively strong degrees of mathematics self-efficacy and mathematics teaching self-efficacy. This is divergent from many findings in the literature (Geist, 2015; Looney et al., 2017) and the researcher's personal experiences. A potential explanation and limitation of the study is that participants with a greater degree of mathematics self-efficacy, mathematics teaching self-efficacy, and preference for mathematics opted to participate in the study. It is quite possible

that individuals who do not enjoy mathematics and those who do not have high degrees of belief in their abilities chose not to participate. Alternatively, it is possible that an increased level of mathematics self-efficacy is present in emerging educators. Further research examining the narratives of participants with more diverse self-reported levels of mathematics self-efficacy and mathematics teaching self-efficacy would provide greater insights. Additionally, research that explores differences in self-efficacy of emerging educators exposed to various mathematic strategies and approaches in their own K-12 experience compared to more traditional approaches would be valuable.

Table 4.1

Mathematics Teaching Self-Efficacy of Pre-Service Teachers

Heather	4.43
Chloe	3.29
Brandi	2.86
Naomi	3.29
Sarah	3.86
Mean	**3.54**

Table 4.2

Mathematics Teaching Self-Efficacy for In-Service Teachers

Jon	3.14
Amelia	3.43
Tara	3.86
Meghan	4.14
Mean	**3.64**

Table 4.3

Mathematics Teaching Self-Efficacy SETMI Results

SETMI Item	Pre-Service Teacher Mean	Standard Deviation	In-Service Teacher Mean	Standard Deviation
Motivate Students	3.20	0.84	4.00	0.82
Help students value math	3.20	0.45	3.75	0.50
Craft relevant questions	3.80	0.84	3.75	0.50
Get students to believe they can do well	3.60	0.89	3.25	0.96
Use assessment strategies	3.60	0.55	3.75	0.50
Provide alternative explanation	3.60	1.14	3.50	0.58
Implement alternative teaching strategies	3.80	1.30	3.50	0.58

Table 4.4

Mathematics Self-Efficacy Pre-Service Teachers

Heather	4.00
Chloe	2.67
Brandi	2.29
Naomi	3.60
Sarah	3.53
Mean	**3.22**

Table 4.5

Mathematics Self-Efficacy In-Service Teachers

Jon	2.87
Amelia	1.67
Tara	2.87
Meghan	3.67
Mean	**2.77**

Table 4.6

Mathematics Self-Efficacy SETMI Results

SETMI Item	Pre-Service Teacher Mean	Standard Deviation	In-Service Teacher Mean	Standard Deviation
Describe characteristics of numbers	3.25	0.89	2.50	1.00
Compose and decompose-add & sub	4.00	0.00	3.50	1.00
Compose and decompose-mult and div	3.20	0.84	2.75	1.26
Teach conversion fraction to decimal	3.00	1.00	2.50	1.29
Teach students to compare equivalent fraction/decimals	3.40	0.89	3.00	0.82
Teach students to interpret inverse relationships	3.40	0.89	3.50	0.58
Teach students to manipulate coordinate planes	3.00	1.00	1.75	0.96
Teach plots and interpret graphs	3.20	1.10	2.75	1.26
Teach area and perimeter	3.20	1.10	3.00	1.41
Convert between units in same system	2.60	0.80	2.50	1.00
Convert units in different systems	2.60	0.80	2.25	0.96
Teach measuring lengths	4.00	0.63	3.75	0.50
Teach discovery of math patterns	3.40	0.89	3.50	0.58
Teach interpret variables in algebraic equa	3.00	0.89	2.50	1.00
Interpret probability	2.80	0.75	1.75	0.96

Themes

Table 4.7

Themes Identified from Research Analysis

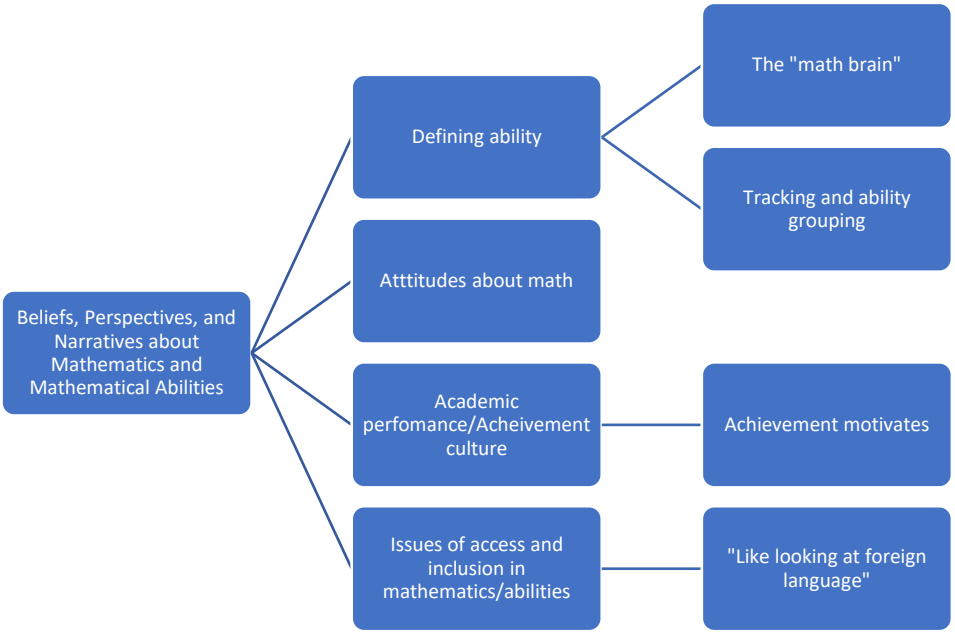
Themes
1) Beliefs, Perspectives, and Narratives About Mathematics and Mathematical Abilities
2) Support
3) Impact of Covid
4) Navigating Curriculum and Practice Disconnects
5) The Influence of Teachers
6) From Students to Teachers
7) Productive Struggle in Developing Math Self Efficacy

Beliefs, Perspectives, and Narratives About Mathematics and Mathematical Abilities. A main theme that came up in nearly every interview and focus group was beliefs, perspectives, and narratives about mathematics as a subject, as well as mathematical abilities. This theme encompassed the beliefs and perspectives of participants, but also views of friends, family, colleagues, students, and even broader societal views. Many participants acknowledged a negative societal stigma when mathematics was brought up in general conversation. As one participant stated,

"I don't think there is any other subject where there is that stigma attached to it." Many participants referred to the commonly heard and societally accepted rhetoric of "I'm not a math person." They also described mathematics through an achievement-focused lens. When determining if they were capable mathematics students, grades seemed to be a key indicator for most participants. These grades served as extrinsic motivators. From an achievement-centric perspective, participants described a view that there are those who achieve and those who do not. Relatedly, mathematical abilities were often viewed as fixed. In speaking of their students, participants described abilities in fixed categories (i.e., high student, low student). Even if participants alluded to believing anyone can be "good at math," which several did, they often would still refer to abilities through a fixed perspective by labeling students or groups of students.

Figure 4.1

Beliefs, Perspectives, and Narratives about Mathematics and Mathematical Abilities Theme Visual Web



Attitudes about Math. Many participants identified negative attitudes and feelings about mathematics, including their own attitudes and feelings, as well as the attitudes and feelings of others. Sarah described times when mathematics "was just not happy." She recalled it was rare to have a fun mathematics teacher who was in a good mood and that mathematics seemed like "drudgery." She remembered in high school that all the mathematics classes were in their own separate building. "Math was separate. Math was on its own. This is where you go for torture hour." Meghan described an anxiety towards mathematics and a fear of being wrong or perceived as "stupid," however, in hindsight she recognized her capabilities as a mathematics student. Amelia also did not feel positively towards mathematics during her K-12 experiences. Jon described being indifferent toward mathematics. He never looked forward to it but did not mind being in mathematics class. All of the other participants spoke very highly of mathematics. They described it as fun and enjoyable. During the focus group interview, pre-service teachers were asked to share what came to mind first when they heard "math." Figure 4.2 is a word cloud of their responses. This group generally had positive words to describe mathematics.

Additionally, during the focus group, pre-service teachers discussed how they thought the general public felt about mathematics. Figure 4.3 is a word cloud of their responses. Although some words were duplicative, the overall tone became more negative. Pre-service and in-service teacher participants recognized mathematics is often stigmatized, and many people see it as "drudgery," as Sarah described her early mathematics experiences. Participants described how our society has normalized negative attitudes about mathematics. In normal conversation, it is acceptable to say,

"I'm not a math person." Being a "math person" becomes an identity, which is not as common in other subject areas. It is less common to hear people say, "I'm not a reading person" or "I'm not a social studies person." When individuals described disliking these subject areas, the rhetoric became less identity-centric. For example, Heather stated, "I don't really enjoy social studies" and Naomi said, "I don't really like reading that much."

Figure 4.2

Words Pre-Service Teacher Participants Used to Describe Math

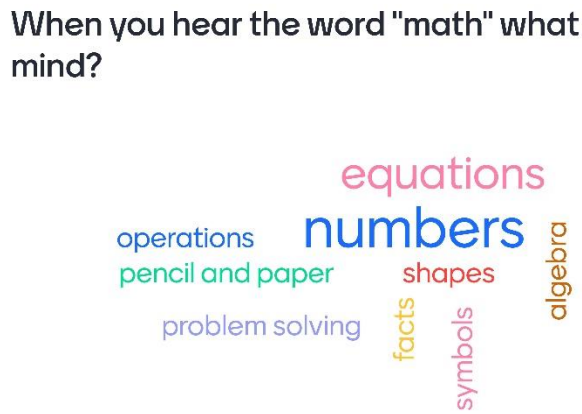
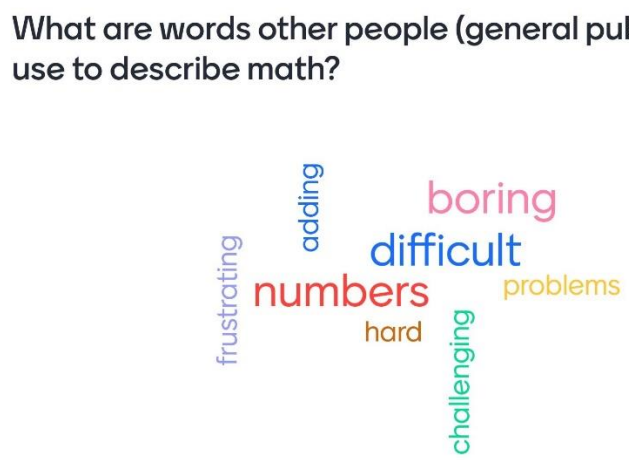


Figure 4.3

Words Pre-Service Teacher Participants Used to Describe How the General Public Feels About Math



Academic Performance and a Culture of Achievement. Many of the participants recalled they performed well in school, regardless of certain academic challenges. Interestingly, their mathematics performance was discussed as the primary indicator of their mathematics ability. Participants described their perceived ability in several ways, "I was always pretty good at math as a kid," "Like math, I'm pretty good at," "I've always been good at math," "I'm good at math," "I'm supposed to be good at this."

In describing his experience as a student of mathematics, Jon used his grades as an indicator of his success and understanding. As he recounted "doing really well" in his sophomore and junior year and he added, "I think I got As both those years." He also described pre-calculus as more challenging but that he was able to get "an A everyone once and awhile on a test or something, quiz." Jon did not mention the concepts he learned, the relevance of mathematics to his life, or how he engaged in meaningful mathematics learning. These were not identified as indicators of mathematical understanding, but the grades were a benchmark that he remembered.

Brandi also discussed her challenging time in mathematics being related to her grades. In high school she remembered, "ending the class with the worst grade" she had ever gotten which impacted how she thought about mathematics. Brandi explained, "This is the first time that it isn't that easy and I'm not loving it." Brandi vividly recalled a teacher who posted her mathematics grades outside the classroom. She remembered having to walk up to a list, find her student ID, and follow her finger across to find out her grade for the class. She discussed how having her "grades on display" made her feel uncomfortable. She thought it would be easy for another

student to determine her grade for the course. For many participants, grades and achievement played a central role in their mathematics learning experience.

Achievement Motivates. A culture of achievement appears to have motivated several participants. Chloe described being validated by grades and feedback on her work, indicating, "the grades encouraged me to do well." Heather highlighted how "in elementary school and stuff, I wanted the sticker on my paper. So, knowing I got a sticker, I knew I did well enough. I know what I am doing." "Good" grades and rewards for meeting or exceeding performance outcomes validated participants' mathematical abilities and motivated them to continue to achieve. Mathematics was less a subject to engage in deeply and more a set of tasks to achieve in order to demonstrate competence.

Brandi even described being less motivated when her high school moved towards a competency-based grading system. She described it being strange to be able to take an assessment multiple times to "get whatever grade we wanted." She reflected that she did not work as hard with the competency-based system, as she had with the traditional letter grade system.

Brandi and Heather also compared their mathematics performance with that of their peers. Brandi indicated she liked mathematics because she was "the best at it" and Heather discussed that being able to understand mathematics "more easily than even others made it more enjoyable for me." Chloe discussed how she was motivated to "do well" in mathematics because of indirect messages she was getting around her that mathematics was an important subject area. She saw mathematics in her daily life and was able to apply it. She also came to understand it was a subject that was valued

by her parents and teachers. Chloe remembered making intentional efforts to work harder in mathematics and science. These were the subjects she was driven to do well in. She considered this in relation to societal views of women and girls in STEM,

> I just tried harder in math and science and thinking back on it, I feel like being a girl and being good at math and science is not, like, the general public's perception. So, I think that motivated me to try harder and do better in those subject areas just because I think a lot of times when people think "oh well, girls aren't necessarily good at math and science" because I think they are very male-heavy fields, especially when you get out into the career.

Sarah's motivation differed, as did her experience with mathematics. Sarah struggled with mathematics in her own K-12 experience and later discovered at community college that she was capable of doing mathematics and being "a mathematician." Because of the barriers she faced she became motivated to help other students understand their own potential to be mathematicians with her:

> I wanted to make an opportunity for me to say, "no, you're invited to be good at math too. You don't feel like you are, but I want you here. I want you to be a mathematician with me. And I had to fight to get here too. So, you can too." I want to inspire kids.

Participants also discussed the achievement-focused mindsets of students. Several participants discussed "right" and "wrong" answers and how students perceived and responded to being "right" or "wrong". Heather differentiated mathematics from other subjects, which highlights the emphasis on achievement:

> I think that when people don't like math, they get frustrated and confused because it's just kind of been like this is the right answer, this is the wrong one. And a lot of subjects, like, pretty much every other subject, isn't like that.

Similarly, Chloe described, "I think there are a lot of kids that they want to be right" she later said, "there is a lot of settings, especially in schools, where it's 'oh, if I'm wrong that I get a bad grade' or something like that which doesn't feel very good, especially for a young kid." Although recent mathematics reform efforts have emphasized mathematics reasoning and problem-solving over a primary focus on computation, teachers may still view computation as a necessary prerequisite for problem-solving (Baker & Galanti, 2017). Therefore, teachers may place a greater degree of importance on memorization and fluency in their mathematics classrooms (Baker & Galanti, 2017) which may cause students to strive for accuracy and find a "correct" answer.

However, Chloe wanted to create an environment in her classroom where students were not afraid to get a "wrong answer" but were more focused on the process. Similarly, Meghan discussed the importance of creating "an atmosphere where they all felt comfortable being wrong" and that mistakes were not only expected but praised. Both Chloe and Meghan emphasized the value of mathematical reasoning in their teaching. However, they described that students did not always readily welcome this shift in thinking about mathematics. Many students wanted to get an answer correct. Meghan described many students being concerned they were "wrong" if she asked them why they came to a certain answer in their work. She conjectured, "I think [it] is a very human instinct, or maybe American instinct, I don't know. If

someone asks you why, you must be wrong." This "instinct" appeared to be a persistent pattern that many participants observed. Chloe tried to engage in mathematics discussions by asking probing questions that encouraged students to explain their thinking and she described having received many blank stares and silence in return.

Many participants spoke of discussing mathematics reasoning and moving beyond seeing mathematics only as computation. Participants described learning to value different perspectives and approaches to mathematics problems. This was different from when participants learned mathematics when the expectation was "one way" to come to a "right" answer. However, as Chloe and Meghan found, this paradigm shift in what mathematics is and how it can be expressed was not always easy to enact in the classroom. Achievement in mathematics is deeply embedded within our culture and it is not readily replaced with broadening perspectives that move away from right vs. wrong.

Furthermore, there are factors external to individual classrooms that may reinforce an achievement culture in mathematics. Even if one of these individual emerging educators successfully deconstructs elements of achievement-centric mathematics practices within their classrooms, there are still school and district expectations that are founded on these grounds. Mathematics curricular programs will be discussed further as a separate theme, but many programs utilize workbook pages to practice computation repeatedly and end-of-unit assessments that are used to make data-informed decisions about students. Participants also spoke about the expectations

to follow the prescribed program and pacing, which they did not always feel was right or best for students.

Chloe recalled from her own time as a K-12 student being frustrated by the lack of feedback she received on her mathematics assignments and assessments. Receiving a paper back with Xs or a "C" did not help her to understand how to improve or grow. She discussed feedback and how she saw it being delivered to students in the school she student taught at:

> I mean right now that's the program we're using, and that's their end-of-unit assessment, and that's what the administration expects to get and see data on. But I think the way it is being taught at the school I'm at now, it's an opportunity to throughout, like, the math module they are getting that constant feedback, and at the end they … The test is like here's your score, but I think prior to that they were able to get that feedback to hopefully be successful on the assessment? I don't really know. I think that there are definitely things I would do differently than this program does it. Um, I think at least they have opportunities to get that feedback in the moment then I guess … take a test? But it's a slippery slope for sure with testing and homework.

Even for emerging educators that wanted to move beyond a focus on achievement, there were barriers and challenges within their educational systems and societal expectations. Although they may come out of their teacher preparation programs questioning current practices, they did not necessarily feel equipped to enact changes.

Issues of Access and Inclusion in Mathematics/Abilities. As participants have described, there are emotions tied to mathematics and many of these emotions become

internalized. An achievement-focused classroom environment and society reinforced the idea that mathematical ability was a scarce resource, that some individuals had an innate mathematical gift, while others should pursue other subjects. Mathematics became a subject that is meant for some, but not others. Mathematics was often described as exclusive, which brings up questions of access and equity. One participant, Sarah, described mathematics as an elite club that she did not feel invited to. Sarah stated,

> I didn't feel like I was invited to the math world because sometimes I feel like people have this way of thinking about if you are good at something…and because I was not naturally gifted, I didn't receive encouragement, maybe? Or I *felt* like I didn't. It was probably internal. Maybe a bit more that.

She brought up a topic that was prevalent among participants: how learners viewed their mathematical abilities and how this was tied to their identity as mathematics learners. Terms such as "good at math", "naturally gifted" and "math person" arose in many of these conversations. Academic performance in mathematics was discussed in a way that made it closely connected learners' abilities and potential as students of mathematics.

Sarah went on to discuss how she had come to understand how mathematical abilities are viewed and understood. She stated,

> You talk to people who are good at math, and it is almost just like someone who knows how to draw and other people can't. You don't know how to explain it and it is kind of like, well just forget it, you don't do this. And not everyone does this to people, but if you don't get it, you don't get it, and we'll

> push you through as far as you can. High school was like that. “Don’t worry about it, you don't need to do anything more.”

These messages of ability and potential that participants received from their environment and other individuals stuck with them. They began to internalize the messages they received. Sarah described internalizing negative societal views of mathematics; “all that stuff that makes us feel like ‘ooooh math. Not-uh, not me!’ was stuff that I internalized.” Sarah was not the only participant to feel excluded at one point or another during her mathematics education.

Jon recalled middle school being the time when mathematics became “tough” and “confusing” to him. He reflected on the other students around him and compared his own capacity to do mathematics:

> I think there were a lot of other students around me who it came a little bit or a lot bit easier to them–some of the concepts we were working on. So, I was kind of noticing that. And it seemed to me that I was the only one struggling in the class and everyone knew what to do and I had no idea. Yeah, which I’m sure wasn’t the case, but it felt like that. So, I think that is why it felt frustrating more to me.

Meghan also received messages from her environment that she described internalizing. In Meghan’s case, her mother dissuaded her from pursuing mathematics while at the same time encouraged her sister to pursue mathematics. Meghan said,

> Growing up, my mom would be like, “Oh, math. Math is your sister’s thing. You do the writing thing.” And my sister would be told, “You do the math thing. Writing is your sister’s thing.” So, you kind of grow up with that thing.

Meghan's story is not unique. Existing research has indicated parental perceptions of their children's mathematical ability in middle school have an indirect influence on students' own perceptions of their mathematical abilities as they enter high school and their desire to pursue mathematics and science careers (Bleeker & Jacobs, 2004). Meghan reflected on this during her interview as she described how an influential professor she had during her teacher preparation program advised teacher candidates to be mindful of how they talked about subjects and to not minimize, discount, or talk negatively about a subject or their own abilities. Meghan recalled prior to taking this mathematics methods course, that she would tell students when she was subbing that she did not like a certain subject area, but after the class, she was much more intentional in speaking positively about subjects and embracing a challenge. Being mindful of how others see themselves as learners was something at the forefront of Meghan's mind because she related to not believing in her own abilities as a mathematician, partly due to the messages she had received.

"Like Looking at a Foreign Language." An access and equity concern that several participants discussed was language and vocabulary used in mathematics curriculum. Jon described the mathematics program at his school being "very language-based." He discussed that students who had difficulties in reading also found challenges with the mathematics program because of the need to read. Existing research supports this observation, suggesting reading comprehension skills are important to students' success in solving word problems (Boonen et al., 2016). Sarah also recalled her own challenges understanding mathematics language that resulted in her taking courses out of the recommended sequence. She recalled, "There were a lot

of vocabulary terms that were used and used in such a way that it was almost expected that you know what this is and what this means already. I wished I had been there with my dictionary just looking it up." As a result of this experience, she described emphasizing mathematics-specific vocabulary in her teaching. For students who are struggling to read or for students whose first language may not be English, it is important to consider the potential barriers that a reading-focused mathematics program may pose in the classroom. Existing research has suggested that students whose first language is not English, may experience mathematical challenges because of language-based mathematics curriculum, and students with language-based impairments or communication disorders may also experience challenges with mathematics tasks with certain language demands (Alt et al., 2014). Students may find themselves struggling in mathematics, not due to a lack of mathematical understanding, but because they lack the comprehension and language skills, or the processing skills required to access the curriculum.

Defining Ability. An interesting observation was the words participants used to describe their students' mathematical abilities. Brandi described students in her class who were "definitely higher kids, ones in the middle, and lower kids." She discussed knowing that some students understood mathematics concepts because they were doing well on their assessments. The classroom she interned in also grouped children based on perceived mathematical ability for an extra mathematics block. Students were grouped as either "low," "medium," or "high" and would work on targeted skills. Brandi described working differently with students during independent work time based on how she perceived their ability. If she believed a group grasped a concept,

she let them go work on their own while she asked students she perceived as struggling to stay with her for more remedial support.

Chloe described a Title 1 tutor who supported "three of our lowest kids" but she also shared a story of how her perceptions of a student's abilities were inaccurate. Chloe completed a cognitive interview assignment in one of her courses within her teacher preparation program. She chose three students of different skill levels, which she described as "higher," "lower," and "in between in math." She recalled being surprised by the results of her interview.

> The low kid did WAY better than I expected which was really interesting. It was interesting because I think that I don't know if it was because he knew it carried no weight. It was more, I just want to see what you know than being in a whole group setting, like oh no, that's not the right answer. Um, there really was no stakes at all so I don't know if that's why or if it was just the way they were able to think about those problems in a more 1-1 setting. I don't know, but that was a really interesting thing I got to do.

Chloe described this cognitive interview as "low stakes" and that it "carried no weight," which is interesting to consider what mathematics activities are perceived as "high stakes."

The school where Naomi student-taught used Kagan structures. She described students at desks in groups of four with a "high achiever," a "medium/high" student, a "low student/below grade level" and a "medium." Naomi explained that rather than grouping students by ability, the Kagan structure allowed students at different levels of understanding and skill to learn with and from one another. This was an exciting

practice for Naomi to see and one she planned to explore using further. Naomi was the only student to describe mixed groupings compared to in-class ability groupings.

The "math brain." Many participants viewed mathematical ability as a fixed trait rather than as a malleable trait. Many participants had views about themselves as learners of mathematics, which may have changed over time. However, they also generally had views of students' fixed mathematical abilities. In describing a student, Jon discussed one student whose "brain works pretty well with math." This suggested a belief that certain individuals have a greater capacity to do mathematics. Similarly, Tara spoke about a spectrum of ability that could be expected in any classroom and normalized that there would "always be some children" who are "typically behind their peers."

Amelia saw herself as "the black sheep of the family in terms of math." Coming from a family with family members in mathematical fields, she always felt different. She said she "didn't quite have the math brain that they did." She saw her family members enjoy and flourish with mathematics, but it was always more challenging for her as a K-12 student. However, she enjoyed using data in her classroom and considered this as "where the math comes into my brain."

Heather presented as confident in her abilities to do and teach mathematics. As she recounted her own mathematics experiences, she had many fond memories of mathematics. It was a "fun" subject area. However, she experienced challenges in geometry and pre-calculus. Heather saw these challenges as a reflection of her ability to do mathematics. She said,

> I started to understand, like, the geometry and pre-calc and different types of math that just aren't for me…I can do parts of them, but, like, I would never want to do all of them. So just coming to that realization I can realize and know I'm not good at those ones and I don't have to like them now.

Heather's struggle in geometry and pre-calculus not only deterred her from these subjects, but the challenge made her reconsider her ability to do geometry and pre-calculus as complete subsets of mathematics. This was not a challenge she was interested in overcoming, but rather the struggle indicated to her this was an area that was not meant for her. Not only did Heather see her own successes and struggles as an indicator of her potential to be successful or not successful in mathematics, but she generalized her experiences to understand how others' challenges and successes must dictate their mathematical ability. She indicated, "I just feel like understanding some things people are better at and some things people aren't great at. And understanding that everyone's that way." This clearly has concerning implications within the classroom. Existing research indicates that teachers who view mathematical abilities as fixed tend to engage in classroom practices that promote fixed mindsets in their classrooms, such as grouping students and setting expectations for students based on past achievement data (Sun, 2015). A fixed view of abilities might be detrimental to certain students in the classroom if they are not provided the same level of support and expectation from their teacher.

Tracking and Ability Grouping. Participants also described how they were put into different tracks of mathematics classes. Brandi recalled being put in advanced-level mathematics courses starting in middle school and starting to struggle in high

school. Brandi was in the "advanced" level class for the first two years of high school but was still struggling so she moved to what she called the "regular" class:

> I took regular pre-calc because I was just like, you know, at this point, I just can't keep up with the advanced. That was a positive experience, but it was almost too easy. I could just tell I was, like, one of the only ones who, like, really understood anything that was happening. So, like, okay, I can't find the happy medium, really. Um, you know? I didn't really feel challenged in that class.

Brandi also differentiated her ability from most of the other students in the "regular" class. She referred to "low kids" and one friend "who was similar to me ability-wise." The "low kids" were struggling, while Brandi and her friend were "bored a lot." Naomi was also put into an advanced-level track beginning in middle school. She described the advanced track as being an accelerated pace that was a grade level above what was expected. She described the fast pace as a barrier and challenge for her. "I think they just expected if you were one above you could just go really fast." Although participants were initially excited to be placed in an advanced class because it felt like a reward for their achievement, once they started to encounter challenges, they felt they no longer belonged with the group.

In-class ability grouping. Many participants referred to small group instruction as a key part of their mathematics instructional time. Additionally, participants described differentiation of mathematics which occurred primarily during small-group mathematics instruction. Many participants described a whole group lesson from the school's mathematics program followed by a small group or

independent work. Overwhelmingly, groups seemed to be organized by perceived ability. Chloe positively described the differentiation in her mentor teacher's classroom. She stated,

> And after that we do math groups and that I think has been really effective because we are able to put kids on level with other kids that are kind of, like, on level with them and we can differentiate the instruction to what meets their needs. Like, the higher kids might not need to do anything with base 10 blocks, like, they've got that now, but what can we do to, kind of like, challenge them a bit and going back to square 1 with some of the kids who are lower.

Brandi also referred to in-class ability grouping as a structured part of the mathematics instructional time: "We had students sorted high, medium, and low math ability and we would work specifically on what they needed, kind of backfilling those concepts." Meghan also described differentiation as a key part of her mathematics instruction that occurred primarily during small group times with students in groups organized around perceived ability. She explained that differentiation was emphasized as important during her teacher education program for every lesson. She recalled the program highlighting the need to "stretch the child so they are getting what they need." However, Meghan described she found it more challenging to do this as part of the whole group lesson because "you have to be more mindful of your time. I can't, in the whole group, wait for my lowest to get all the concepts." Again, in discussing differentiation, students were often labeled based on overall perceived mathematics ability.

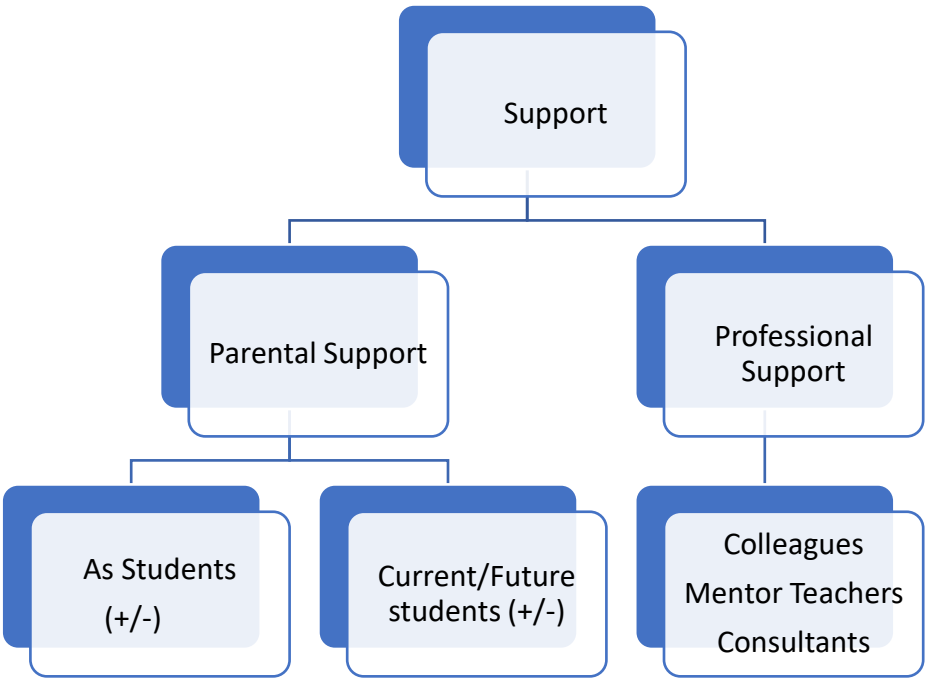
Sarah also described challenges differentiating instruction at the whole class level. She mentioned the need to keep the class together: “the herd has to kind of stay together in my mind in math.” She said that she would have an extension activity ready for those who are ready to go on, but this was particularly for those she perceived as readily mastering concepts. There was little to no mention of differentiation and modifying access to high-quality mathematics tasks for all students. Chloe mentioned “low floor, high ceiling problems,” at one point but did not elaborate on how she might deliver a task that could provide an entrance for all learners and the potential to expand to higher levels for students who readily grasped the concepts. It seemed that participants received messages that differentiation was important to do to meet the needs of students but to enact differentiation within their classrooms, many participants split students into small groups based on perceived abilities. Participants described challenging their “high groups” with extension activities, further questions, and greater tasks while providing extra support and additional fluency practice for “lower kids.”

Considering motivation in this context, it is reasonable to wonder whether those students in the “low group” were less motivated given the mundane mathematical tasks, while the “high group” may have been motivated to achieve and rise to the challenges presented to them. Further research from the perspective of the students within this local context would be valuable. Existing research has indicated that well-intentioned differentiation practices can inadvertently create inequities within the classroom (Anthony et al., 2019). Fitzgerald et al. (2021) argued there is a need to move away from in-class ability grouping practices and embrace complex

relational and responsive pedagogy. Similarly, Anthony et al. (2019) argued prioritization that focuses on "student well-being and productive mathematical disposition" over labels that reinforce achievement outcomes (p. 117). Fitzgerald et al. (2021) found a change in beliefs about students' abilities challenging. Therefore, teacher educators can consider opportunities for pre-service teachers to experience "disorienting dilemmas" that question in-class ability practices and present complex relational and responsive pedagogy as an alternative.

Figure 4.4

Support Theme Visual Representation



Support. The majority of participants spoke of an individual or individuals who supported them at least one point in time as they were learning or learning to teach mathematics. Several participants spoke about their parents providing support to them during their childhoods, most notably while doing homework. Additionally, participants spoke of the support they received from colleagues and other staff

members as they were on their path to becoming educators. This support was meaningful because it enabled participants to struggle, but they were still able to persist through their challenges.

Parental Support. Parental support was mentioned by several participants. Jon, Heather, Chloe, Naomi, and Amelia all mentioned their parents supporting them with mathematics homework when they were younger. Jon, Heather, and Chloe shared a very similar memory of sitting at their kitchen table doing mathematics homework with their fathers. All three participants experienced frustration because they were confused by their mathematics homework and their parents were unable to assist them. They described their parents as capable in the area of mathematics, but tension would arise because the parents had learned to represent mathematics in a different way. Jon and Chloe remembered tears at the kitchen table and Heather recalled eventually doing mathematics homework on her own without her father's help. Naomi's father would provide her with supplemental mathematics work. She recalled that each summer when school was out, her dad would announce that it was time for the "Dad Homework." Naomi also recalled that her father took a different approach to mathematics. She remembered learning lattice multiplication in school and her father teaching her the traditional method of multiplication instead, telling her the lattice method was "just confusing" her. Naomi adapted and learned both methods. She would do lattice at school and the traditional method at home.

Naomi also remembered her parents took an interest in her mathematics achievement at school. In high school when she faced some challenges in mathematics, she recalled her parents driving her to school early to get extra help in

mathematics. When Amelia started to struggle in mathematics in fifth grade, she remembered it was a “big deal” for her family. Her mother scheduled a meeting with the teacher. Another “vivid” memory she shared was a particular mathematics homework problem that stumped her mother. She recalled her mother waking up in the middle of the night to call her grandfather to try to solve the problem. Her father would also request a copy of the mathematics textbook’s teacher manual to be able to best support Amelia with her mathematics work at home. He would create study guides and sample problems to help her.

Chloe remembered that her parents would take opportunities to integrate mathematics into daily activities. She recalled going out to the store with her parents and they would ask her how much money they would need at the checkout with the items in their cart. She remembered seeing the need for mathematics in baking and other areas of her life. She felt that her parents placed importance on mathematics.

Naomi also discussed getting her homework checked by her father every night after she completed it. Her father would help her to correct any errors. She recalled arriving at school the next morning with her homework corrected and knowing where she had gone wrong, while her peers who had independently completed homework “to the best of their ability” were often unaware of their errors. This gave Naomi an advantage and essentially private mathematics support at home.

Parental support, even when frustrating, provided each of the participants with an extra layer of support. Interestingly, this level of support at home was something that many of the same participants identified as lacking with students they had worked with during student teaching or in their classrooms. The lack of parental support or

effectiveness of parental support was something that was identified as a challenge and a barrier. The parental support piece had become the most apparent to participants in relation to homework.

One challenge, similar to what some participants experienced with their own parents, was that students were learning to use different approaches and strategies that were divergent from how their parents learned. Participants described parents being unable to understand and support their children with mathematics homework. Chloe described receiving homework back from a student with a note saying, “An adult did not know how to do this” with a smiley face. Tara had engaged in many conversations with parents who were confused by strategies or who had negative attitudes toward mathematics. In addition to parents not understanding and being unable to effectively assist their students at home, participants also discussed a lack of any parental support at home to assist with homework. This was a challenge, particularly for students who were struggling to grasp mathematics concepts. Naomi described the homework dilemma this way: “So it’s like how do you get them to where they need to be when they are not going to have support at home? How do we maximize the time at school with them?” Chloe believed learning does not stop once the school day ends and that it continues at home, as it did for her when she was in school. Mathematics came alive for her at home, as she saw applicability in daily life. However, she questioned the value of sending homework home because parents were not always available or able to support children. She explained,

> There’s really no point in sending homework if they are not practicing the skill the correct way, which isn’t always the most important thing, like, I think if

> they can add, that's important, but if we are learning to make a 10, I don't think it's useful for them to go home and do a make a 10 homework page a different way.

Existing research has suggested that parental support and involvement in students' learning yields benefits for students (Barger et al., 2019; Wu et al., 2022). However, a recent study suggested parents and elementary-age students have more negative experiences with mathematics homework at home than positive experiences (Wu et al., 2022). Furthermore, when parents had higher levels of self-efficacy, they were more likely to help students with mathematics homework (Wu et al., 2022). When parents were not involved and had a negative outlook towards mathematics, this could have an impact on whether their children enjoyed mathematics, persisted during challenging mathematics, and met academic outcomes (Wu et al., 2022). In addition to Chloe's concern about students practicing different strategies at home, there were also concerns that negative parental attitudes and lack of parental availability and assistance may have detrimental consequences for students.

Parental support can be considered from an equity standpoint. Many of the participants of this study benefited from parental support as they grew up. Even with minor disagreements and frustrations, most parents were supportive and available to assist with homework or advocate for their children. In contrast, participants saw parental involvement or lack of parental involvement as a barrier for their students. Children who had parents at home who were there to support them with homework were at an advantage over those who did not have someone at home who could support them with mathematics homework. Additionally, children may be at different

levels of increased advantage depending on the mathematics knowledge of their parents or the willingness of the parent to embrace different mathematics strategies children were learning at school. Naomi successfully navigated between the methods she was learning at school and the traditional approach her father taught her at home. For others, such as Jon, Heather, and Chloe, the different approaches were confusing and frustrating to them and their parents. However, they still had support at home. Those who did not have someone at home to help were left on their own. They came to school with their homework incomplete or incorrect. As Naomi pointed out, she was at an advantage knowing she did her work correctly because her father had checked her work, whereas other students might have been starting their day behind their peers simply because they did not have an adult at home to check their work.

Professional Support. Another important source of support for participants was their professional support networks. Many pre-service teachers spoke highly of their mentor teachers as a source of support they could turn to and ask for guidance. Additionally, both pre-service teachers and in-service teachers discussed the benefits of connecting with other colleagues on the grade-level team. They could plan mathematics lessons together and discuss concerns. Brandi emphasized the value of her team, stating, “So my whole team, the second-grade team, is really close. We talk about math all the time.” Additionally, several participants mentioned having access to a mathematics specialist who would be available for consultation or even co-teaching. Amelia’s district had partnered with an external mathematics consultant who had been providing district-wide professional development. This consultant was incredibly helpful in supporting Amelia build her capacity as a mathematics educator. The

consultant introduced different strategies and modeled mathematics discussions for staff. However, the most exciting aspect of the support for Amelia was the applicability of what the consultant had to offer teachers.

Meghan discussed being grateful for the supportive staff she works with. She often sought advice from her teacher-partner and other colleagues to gather insights into teaching practices and strategies that have worked for veteran teachers more familiar with the curriculum. Meghan also found it beneficial to plan with her teacher partner at her grade level. Similarly, Jon found it helpful to be physically located next door to his teaching partner at his grade level. He described,

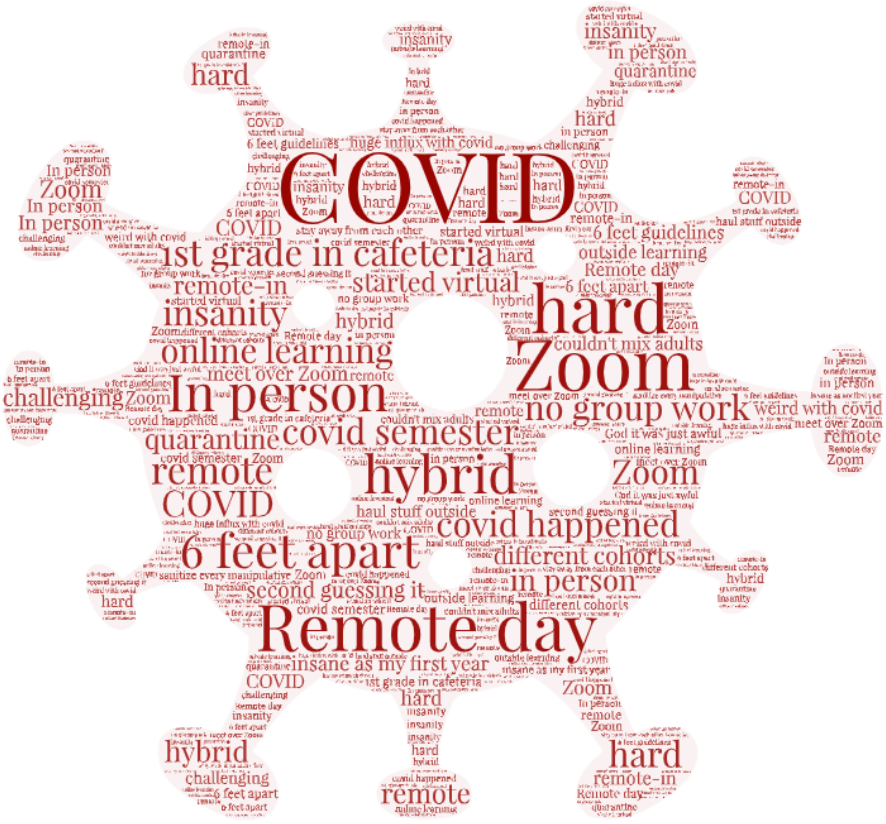
> Throughout the day I'll have time and my teaching partner is just right next door. So, I can just open my side door if I have a question. Like, "hey, what do you think about this? Real quick, let me ask you 'blank'."

Several of the in-service teachers spoke about the value of connections with colleagues, but also of the initial limitations of collaboration with colleagues as they started their roles during the Covid-19 pandemic. Because of the global pandemic, restrictions limited in-person meetings and opportunities to engage with colleagues during meeting times or informal conversation. Tara spoke of starting to teach in her own classroom during the start of the Covid-19 pandemic and how difficult it was to not have colleagues to turn to for support. She described, "You couldn't mix adults. So, there was no talking to colleagues because everyone was locked into their little…you couldn't leave your kids." Amelia discussed wishing she had more time to discuss curriculum changes and competency-based grading with her colleagues, but due to the demands resulting from the global pandemic, there had not been time or

opportunity to engage collaboratively with her grade level the past few years. Jon also commented on the hope to have more time to work with colleagues, but this time had been challenging to find.

Figure 4.5

Visualization of the Impact of Covid-19



Note. This figure is a visual representation of participants' words describing teaching and learning to teach mathematics during the Covid-19 pandemic.

Impact of Covid-19. The Covid-19 pandemic was brought up during nearly every discussion with participants. In some instances, Covid-19 disrupted participants' student teaching and teacher preparation coursework. Other participants began teaching during the pandemic and did not experience a typical start to their teaching careers. Teachers faced challenges instructing and assessing mathematics learning remotely, while others balanced in-class challenges with quarantining, social distancing, and restrictions on group work. Additionally, some participants spoke of learning gaps and behavioral challenges they attributed to the effects of the pandemic.

Figure 4.5 provides a visual representation of participants' words as they described the impact of the Covid-19 pandemic on their experience becoming teachers of elementary mathematics.

Pre-service teachers discussed various impacts on their own teaching preparedness. Heather described how the pandemic impacted her student teaching internships. Due to the pandemic, her full-year internship was split into half semesters, and she was only able to go to the school once per week during the first half of the year. Due to the reduced time in the classroom, Heather felt like her "internship wasn't really like what our internships should have been." Jon shared a story from his teacher education program of when he was reconsidering teaching as a career altogether. This was in part due to learning challenges he experienced himself during the pandemic which occurred during his student teaching semester focused on mathematics and science. He described struggling with the shift to learning online and that one of his faculty members wrote a letter to the Dean to place him on an "academic watch." Jon recalled,

> When Covid happened, it was just (*pause*) and for me, the online learning was not my strong suit. I didn't do well with it. I didn't enjoy it at all. It was a lot more difficult for me because it was like okay, now you are going to basically teach yourself, kind of. I mean, you can have Zoom and everything, but your professor is just sending you stuff to read or to work on and then you talk about it over a Zoom call, and then that's just it. That's kind of where my professor moved to, "well you need an extra little push of motivation if you don't…" I forget the wordage. "If you don't keep up and turn stuff in that is

> appropriate." Just turn stuff in that is, I guess, "up to standards" and "on time" then like, then I guess your time in the program is on the ropes. Which I don't know. I guess it was valid. I mean, just like the online learning was hard and the situation in general for everyone was hard. And so yeah, I don't know if I did particularly well with it obviously.

Jon made it through the semester and successfully finished his program. The following semester he was placed with a mentor teacher for student teaching that he credited as influential to his success and renewed passion for the field. The following semester "sealed the deal" for Jon that he was on the correct path and had what it took to be successful.

Throughout interviews and focus groups, participants shared various stories of starting their teaching careers during the Covid-19 pandemic. Participants described becoming a teacher during the pandemic as "insane," "hard," and "challenging," and shared specifically their challenges teaching and assessing mathematics during this time (see Figure 4.5). Several of the in-service teacher participants with only a few years of classroom teaching experience had taught in various mediums including completely virtual teaching, hybrid teaching, and in-person teaching. Amelia described it this way:

> We started virtual, then we went hybrid where I had two different cohorts that came on opposite days and we would just give assignments on their off days. And then we came back in person the last six weeks of school. I had the full experience.

Amelia specifically discussed how challenging it was to assess her incoming first graders' knowledge of mathematics remotely over Zoom and to utilize manipulatives to support a concrete understanding of mathematical concepts. She could not circulate among students, see their work, or provide guidance the same as she would have been able to in person.

Tara's school remained in-person for the majority of the pandemic, but she still discussed challenges when students needed to quarantine and would need to "remote-in" to a lesson. She described not only trying to provide the mathematics lesson to her in-person students and remote students, but also to the remote students' parents. She described, "And so then you turn into teaching the parent math, you know? And because they are little, the parent is with them obviously. The parents tend to be right there while you are going over the lesson." Tara described additional unique challenges of teaching mathematics during the pandemic. She reflected that:

> Last year we were outside a lot, right. So, we had a tent, and the kids are in camp chairs. I am teaching kindergarten. We have like a small golf course. So, on the golf course under a tree because we didn't have tents yet, while the kindergarten kids are in camp chairs with clipboards, teaching them how to do math. It was a year. I mean I would haul… Everyone had their own manipulatives so everyone had their own little box of stuff. So, I had a wagon and haul all the stuff outside with my portable whiteboard. Honestly, looking back I don't even know how they learned anything. Like, insanity now.

In-service teachers had to demonstrate adaptability in their teaching as they were gaining their first experiences as classroom teachers. Tara was not the only in-service

teacher participant to describe challenges teaching mathematics in person due to Covid-19 restrictions. Meghan, Jon, and Amelia all discussed obstacles they faced as well. Several participants described six-feet space guidelines that prevented them from utilizing small group instruction and collaboration in the classroom. Jon discussed this as a barrier to learning, stating, "You learn a lot working in groups and talking with other people. Bouncing ideas off and a person thinks of something this way and it sets off a light bulb for another person. So that was tough." Tara reminisced how challenging it was to implement restriction guidelines that contradicted what she would normally encourage her kindergarten students to do in class. She reflected,

> God, it was just awful. I wish those poor kids never had to do that year. It was just rough. I spent the whole year yelling at kindergarteners to stay away from each other and don't share. You know, all the things you never want to teach kindergarteners.

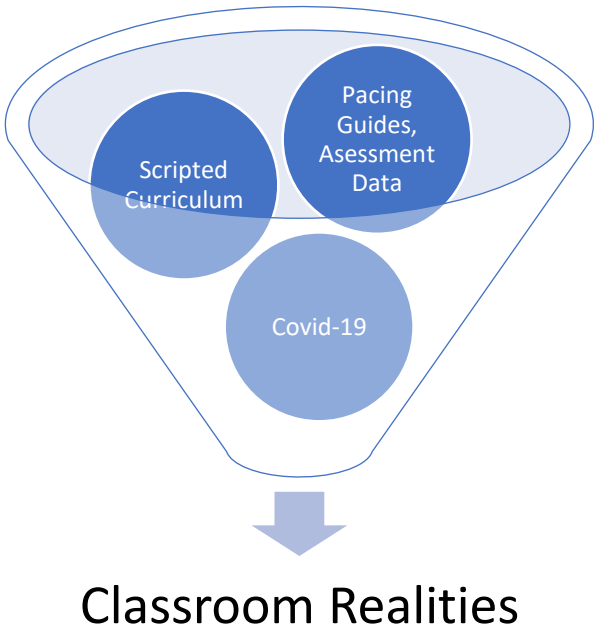
Meghan also described how difficult it was to not have group work in mathematics. This past academic year, she described her third graders wanting to play with mathematics manipulatives and she believed this is due to a lack of exposure and use of manipulatives in earlier grades. Jon also described his school's policy to sanitize manipulatives after each use, which was easier to do with larger tools, such as rulers, but more difficult to sanitize smaller tools, such as base-10 cubes. Each of the emerging educators demonstrated persistence and flexibility as they began teaching mathematics during disrupted school years. Meghan characterized her initial teaching experience as "baptism by fire" and summarized her experience as, "I began teaching in the pandemic. So, it's been a really big test. If I can do these past two years and still

want to come back, this is the career I was supposed to have." This sentiment was echoed by nearly all participants.

These participants provided unique insights into their experiences learning to teach and teaching and assessing mathematics during the Covid-19 pandemic. Many missed out on typical learning opportunities during teacher preparation, such as opportunities to be in the classroom with students and opportunities within their schools as new teachers, such as in-person trainings and collaborations with peers. Another layer to their emerging roles was navigating the complexities of restrictions and unknowns. They had to remain flexible to change and teach in various mediums. Nearly all participants expressed hope for a "normal" school year ahead. They did not indicate feelings of discouragement or being burnt out. On the contrary, their resiliency and passion for teaching was apparent.

Figure 4.6

Navigating Curriculum & Practice Disconnects Theme Visualization



Navigating Curriculum & Practice Disconnects. Another theme from the research that participants discussed was learning to grapple with what was expected of them while weighing what they learned to be best practice and best for students. This often created a practice disconnect. Participants spoke about curricular expectations in their schools, particularly purchased mathematics programs that they were expected to utilize for mathematics instruction in their classrooms. These programs took time for participants to unpackage and learn. This was complicated due to the challenges presented during the Covid-19 pandemic.

Covid-19 shifted priorities and created barriers to planned initiatives. Many of the schools that in-service participants worked at were engaged in curricular and assessment changes. Jon and Amelia's schools had recently shifted to competency-based grading while Meghan and Amelia's schools had adopted new mathematics programs. Amelia described it as challenging starting her position in her district without having time to work on competencies for students. She explained,

> With Covid and with competency-based education being adopted that year that Covid started, that was the first year that had really done competencies. I think the biggest disconnect has been that we haven't really been able to work with my co-workers or other teachers and really look at the competencies and find out how we really want to…what it means for students to really master that skill to show that they are proficient, or extended, or that they are developing. So, and for me, that is the most frustrating thing because that's the time that, I'm coming into something where I don't necessarily know what it means to be

> proficient and no one can really tell me. So, it's hard to find ways for students to show me that.

Meghan also described a "big disconnect" related to the Covid-19 pandemic. She felt that at the beginning of the school year following the Covid-19 shutdown, teachers at her school received reassurance from school administrators that they would have time to "catch up" on lost learning. However, Meghan felt they were not given time to fill in missing gaps and were still held to pacing guides that did not allow for flexibility in meeting students' needs. Meghan's grade level decided to supplement the curriculum through morning mathematics practice, yet they did not readily share this with the school administrators. She described, "That repetition we saw really helped them retain the skills, but at the same time, we didn't tell our admins we were using it because it was outside the math program."

Amelia also described that there was an expectation at her school to follow the prescribed mathematics program closely with few deviations. She described an external mathematics consultant hired by the district who suggested they modify the curriculum pacing guide and move units around. Amelia's grade level spent an entire meeting rearranging the curriculum to align with the consultant's recommendations and thoughts around different times of the year. However, the district's internal curriculum director denied the plan to modify the pacing guide and order of units. The tension between mathematics program expectations and pacing and what participants saw as the best interest of the students was a recurring topic of conversation. Meghan described pressure to move through the mathematics curriculum to keep pace with the

program's guidelines. However, she felt that there were some areas that her students needed to spend more time with to fully grasp concepts. She recalled that,

> Our multiplication/division chapter had so many concepts. I told the consultant I thought it should have been at least two chapters instead of one big honking one. So, this is one of the four basic functions. Like, we can't rush through this.

Other participants shared concerns regarding the curriculum pacing and the reality of what their students were able to master. Brandi discussed gaps her grade level team saw during her student teaching. Her grade level team wanted to "backfill" concepts that students were not proficient in or spend more time on certain units if students did not demonstrate proficiency. However, the mathematics specialist at her school encouraged the team to follow the curriculum pacing guide because the curriculum would spiral back to cover the content at a later time. Adherence to pacing guides was mentioned by several participants as an expectation within their schools.

Existing literature on the use and perceptions of post-Common Core adoption of mathematics curriculum programs is an area of research not readily available in the literature. In one study (Blazar et al., 2020), it was reported that 94% of elementary schools across the United States were using mathematics curricular programs determined by their school districts. This study revealed variations in strict adherence to the curriculum. Although the majority of teacher respondents indicated they use the program frequently, participants often indicated the need to supplement lessons with other materials, and only 8% of teacher respondents describe using only the mathematics program (Blazar et al., 2020). Just as Meghan and Amelia described their

need to supplement based on what they observed with students, teacher respondents in Blazar et al.'s (2020) study determined the need to supplement based on their perceptions of curriculum difficulty. Barrett et al. (2017) found variations in how scripted curriculums were utilized by districts, with some who adopted with fidelity to the pacing guide and contents, while others were more flexible with adaptations. Teachers from schools with strict adoption expectations found it challenging to successfully teach all content. Some needed to cut out other subject areas and were unable to provide differentiated instruction to students, support students who lacked prerequisite skills or serve those who would have benefited from further enrichment (Barrett et al., 2017). With curriculum programs being created to be used across states to match National Common Core State Standards in Mathematics (CCSSM), it is worth analyzing the benefits and drawbacks further. Although these packaged curriculums have been created to address CCSSM and pacing guides have been created to fit grade-level standards into an academic year calendar, there may be local and school-specific factors to consider.

Participants also discussed the increased learning gaps observed in incoming students, which may have been exacerbated by the pandemic. However, the expected pacing of the mathematics curriculum remained unchanged. Meghan discussed her administration expressing understanding at the start of the school year, "Don't worry we know with Covid, and the remote, and the revolving door of who was in school and who was not, you'll have time to catch up." In reality, she felt they were not provided the time to catch up and teachers were expected to still meet the pacing expectations set forth by the curriculum. She described,

> I definitely feel like if we can slow down and get them with these concrete fundamentals, they'll go faster later. But I kind of feel like with some of this pacing, it goes too fast. So, we kind of have more struggling mathematicians than those who are getting it.

However, Meghan was hopeful that her administration would recognize the need to adapt the pacing to meet students' needs. She described a professional development workshop at the end of the school year, where the facilitator advised participants to "ignore" pacing guides. Meghan's principal was present during the workshop, and she was hopeful that "more of that message can come back." Pacing guides created by textbook companies may require further scrutiny if they do not account for students who may not be meeting grade-level expectations, particularly at the start of the year. Students may be at a disadvantage if they receive curriculum instruction at levels beyond their current skill level and understanding.

Another interesting, related topic of discussion that came up with the in-service teachers during the focus group was how mathematics learning is assessed and how assessment data is used. Participants discussed their schools' state testing data, as well as beginning, middle, and end-of-year summative computer-based assessments. Additionally, Amelia, Jon, and Meghan all used the end-of-unit assessments provided by their curriculums to assess students at the end of units. They described many of these assessments consisting of a majority of multiple-choice type problems. Amelia's school utilized beginning-of-the-year computer-based testing to decide which students were eligible for intervention services. She explained this was often frustrating because she was unable to see students' responses to assessment items and she did not

always agree with which students were identified for services through the data. She described one student entering intervention because data indicated he was eligible, only to be shortly released after because "he didn't need it." Amelia did not find the data from the assessments useful to her understanding of her students' mathematical knowledge and understanding. She gained the most valuable information from formative assessments through end-of-lesson exit tickets, observing and taking notes as students worked, or designing tasks for students to complete that align with mathematical competencies. Meghan and Jon also saw value in formative assessment and understood everything in the mathematics classroom to be "technically like an assessment."

Many participants discussed that it took time to learn the mathematics programs their schools used and expected teachers to teach from. Most participants had little prior exposure to learning about mathematics programs before being in the classroom, either during their student teaching or classroom positions. Participants discussed the learning curve associated with learning a new mathematics program and several commented on how they wished they had previewed or seen a mathematics manual during their teacher preparation coursework. Jon described how,

> We never looked at a math manual and I don't know if they have access to it, but I never even looked at a math manual until my last semester teaching, my last student teaching semester. I never looked at a math teacher's manual, or social studies, or science manual, or Fundations, or anything like that. Which I would actually, that would have been something I would wish would have

> happened as well. To take a look to just see what it looks like, see just a general, like okay, this is how it's laid out, the language that is used in it.

Amelia also echoed this sentiment. She wished she had had the opportunity to preview and critically reflect on different mathematics programs during her teacher education program. Amelia even suggested a group observation model, in which it would have been valuable during her pre-service coursework to go in and observe teachers using different curriculum programs and then debrief this within her methods courses. Brandi described reading the mathematics lesson from the manual and being able to implement it as "on-the-job learning." She wished she had had more guidance on how to read the lesson, identify "broader skills," and know how to best prepare to teach a prepared lesson from a mathematics program. Similarly, Tara described reviewing the mathematics manual the night before to better understand what she was expected to teach the next morning.

Other participants wished their teacher preparation programs discussed how to supplement the given curriculum. Brandi described needing to supplement and modify lessons during her student teaching and she would have benefited from this topic in teacher preparation with examples of how to do this. Naomi also brought this up as something she intended to do. She discussed plans to use the mathematics program's curriculum but put "a little bit of" her "own spin on it." Tara began by saying, "Even working with a program, even if it's not *the* program. I mean, schools change programs all the time, you know? Kind of seeing that more because you don't have a lot to that." She went on to say,

> Obviously, once you get into a school, using whatever that school uses. And I think that would have been helpful. Kind of more experience with that and even using it in the classroom somehow, in the college classroom, you know, how to supplement when you don't have. Because those books, sometimes it is great, sometimes it's not. None of them are perfect. And they don't, I mean it can't. It's cookie-cutter. It can't fit every need of every kid.

Meghan was the only participant who mentioned she felt she had adequate prior exposure to a mathematics program curriculum before she began classroom teaching. Meghan gradually learned the curriculum through her subbing roles, and during her teacher preparation program she was required to teach a lesson from her school's mathematics program during her mathematics methods coursework. Existing literature has suggested that even in-service teachers have limited training and professional development in using mathematics curriculum programs (Blazar et al., 2020). In a study that surveyed nearly 1,200 teachers from 345 schools across the country, teachers reported an average of 3.4 days of training on using their mathematics programs and textbooks throughout their entire teaching careers (Blazar et al., 2020). Limited opportunities for professional development and instructional mathematics professional development were also discussed by Baker and Galanti (2017). They argued that professional development should be targeted and individualized to meet the unique needs of educators (Baker & Galanti, 2017). Although comfort levels may increase with the use of the curriculum program, as in Meghan's case, it may be appropriate for schools and districts to create continued learning opportunities for staff training and professional development of curriculum programs. Ongoing professional

development that addresses educators' current comfort level and content knowledge, as well as addresses potential barriers and constraints with scripted programs appears to produce promising results (Baker & Galanti, 2017). This may be especially valuable to determine the benefits and challenges associated with program implementation, address issues teachers observe in how students are responding to program content and provide further professional development in mathematics content delivery for teachers.

The nature of the curriculum that districts expected teachers to use was something that also came up in conversations with participants. Many participants received K-12 mathematics instruction that was guided by a mathematics program and many of them expected to teach in districts where mathematics programs are used. These programs were often similar in their structure and consist of a whole group lesson followed by independent practice. Most pre-service teachers reproduced lesson structure based on what they experienced as students and what is reinforced in the programs used by their schools. Some participants even commented that they were teaching with the same program they learned from as elementary students.

Although analyzing mathematics program specifics is beyond the scope of this research, there are several considerations warranted based on how participants described the nature of the mathematics program they used in their classrooms. NCTM (2014) outlined eight effective mathematics teaching practices that may prove challenging to execute using a mathematics program as described by participants. Within the confines of a scripted curriculum, it may be more challenging to achieve some of these practices such as "facilitate meaningful math discourse," "pose

purposeful questions," and "elicit and use evidence of student thinking." Some participants described reading and following the mathematics manual in their lesson delivery. Although a few participants emphasized the importance of mathematical reasoning, it was unclear how they used mathematics discussions to promote mathematics reasoning. Given the pressures of pacing guides and the curriculum structure that includes both whole group instruction and independent practice, there may be limited opportunities for discussions and opportunities to pose questions to assess reasoning as a result. In describing "use evidence of student thinking," NCTM (2014) described modifying instruction continually to support and meet the needs of learners. Again, with pacing guides and pre-designed lessons and units, it is unclear how readily teachers are able to adjust the content. Since several participants noted hesitations from their building leaders and mathematics specialists to change the curriculum order, it is unlikely that adjusting the mathematics curriculum is encouraged even if students are falling behind.

Amelia discussed not being fully prepared to teach mathematics as she exited her teacher preparation program. She described that,

> I didn't come out of my class and get into my classroom and know what an experience to build their understanding was, and I didn't really have a vehicle for, like, students to talk about or really reflect on their thinking. And so, I think that's where I struggled last year. Like I wanted to have all these experiences for them to build these and have manipulatives they can use to help them understand with a program I didn't quite understand.

Participants often experienced a similar dilemma of how to take what they learned in their mathematics methods courses and make it transferable once they were in the classroom. Participants described spending a great deal of time writing lesson plans during their mathematics methods courses. Amelia recalls, “I just spent hours writing lesson plans which was challenging” and Heather stated, “I just feel like it was a lot more learning like how to write things. Like write the lesson how to do that than even learning about the subjects of things.” Many participants commented they did not often use what they learned in their mathematics methods courses in either their student teaching or beginning classroom teaching experiences. As Amelia recalled, “I can design these cool lessons and activities for this one day or this one hour, that would kind of fit in with your typical teaching, I guess, or your program.” However, she felt a disconnect between learning to create one mathematics learning experience vs. what she could do in her practice consistently.

Once participants entered classrooms, they were expected to follow the designated mathematics program’s curriculum. These programs provided a written curriculum with planned activities, sample problems, and a script for the teacher to follow. Writing a mathematics lesson plan was not a skill that any participants found valuable or transferable to their roles in the classroom. Brandi described that,

> With it all being laid out, it’s not like you learn the content and then decide what to teach. It’s like, no, you just read the lesson. In some ways, it doesn’t promote as deep of an understanding, on the teacher’s part.

Brandi discussed how having a scripted curriculum did not “leave any room for background knowledge.” Many participants felt the learning curve in the classroom

consisted of learning the mathematics program's curriculum. It was challenging for participants to understand how what they learned in their teacher preparation programs was relevant to teaching mathematics within the confines of their district's mathematics programs.

There is a potential opportunity to have pre-service teachers observe programs and decompose lessons (Grossman et al, 2009) within given programs. They may be able to critically decompose lesson units or lesson plans within popular curricular programs.

The Influence of Teachers. Participants spoke of the influence teachers had on their experiences learning mathematics. They spoke of positive experiences with teachers, as well as negative experiences with teachers. Both positive and negative experiences were memorable to participants.

In remembering mathematics teachers with whom they had positive experiences, many participants recalled how the mathematics teacher made them feel. Brandi described a teacher she had a positive connection with:

> I felt like he just valued me and as a student, and thought I was smart, and helped me out when I needed it, and was just better at explaining things, and it was just (pause) we had more of a positive relationship. He definitely let me know he cared.

Jon also described a teacher from high school whose class he looked forward to. In describing this teacher, Jon said, "I think she just made an effort to get to know everyone on a personal level and she made it really clear that she cared you did well. She wanted you to succeed." He went on to say, "She just took an interest in

everything and everything every student did." Jon described being on the basketball team and this teacher taking a genuine interest in the games and even attending the games. This connection was meaningful to Jon at the time, and reflecting on it now as a teacher, he recognized, even more, how valuable those efforts to connect were for him at the time. Math was enjoyable for Brandi and Jon in these classes and these teachers remained in their minds because of how they made them feel. They felt cared for and that their teachers believed in them.

Creating a safe learning environment was something Tara also described. She described a college professor who had a "style" that resonated with her and other students. She described the inquiry-based approach to teaching helped everyone feel "more secure," "more confident, and willing to try and share." Chloe also discussed her mathematics methods professor. The class was "eye-opening" for Chloe, and she described that her professor "made (her) want to be a better mathematics educator." Chloe was not the only participant who described being motivated by a mathematics teacher to do better. Jon also described that he "really liked the teacher and really wanted to do well" in the class with the high school mathematics teacher he had a positive connection with. These teachers that participants recalled created safe learning environments that encouraged participants to engage more fully in their mathematics learning.

Other teachers that participants spoke highly of made mathematics accessible to them. Sarah described a professor in community college who "made the abstract concrete." She recalled, "He was able to take something that was abstract and out of reach and turn it into a concrete format that I could follow." Some participants spoke

of engaging ways mathematics was presented by their teachers or how it was made applicable to their lives. Heather recalled that her statistics teacher utilized real-life problems rather than relying on pre-determined numbers to calculate from a textbook. She thought this made the lessons more interesting and exciting to learn about.

Conversely, participants also described negative teacher experiences when they had limited or poor connections with their mathematics teachers. Other participants described their teachers teaching too fast or not presenting mathematics effectively. Brandi had enjoyed mathematics and felt she excelled at it until she reached high school. She described having a teacher who was also the football coach, who seemed as though coaching was his priority. Brandi described feeling as though this teacher favored the boys in her class, and that she did not have a positive relationship with him. This became even more problematic as she began struggling with the content of the class. Brandi described that,

> If you weren't struggling, I think it was okay. But when I went for that extra help, he could tell I was trying. I was there *every day,* and he still didn't give me that respect, really. So, that was frustrating, like, you can see how hard I'm working. That still didn't make him like me. I didn't feel like he liked me. So that (pause) It's always hard to learn when you feel like your teacher doesn't necessarily like you.

Brandi described this as the first time she was not "loving math" anymore. This was in part because she was struggling with content, but also because she felt as though her teacher did not like her and was unable to help her. Jon did not share any stories of specific teachers he had poor connections with, but he contrasted the positive

relationship he had with his high school mathematics teacher with other teachers. He described it this way:

> I think some teachers, I guess didn't make that effort, or it wasn't as pronounced, maybe. And so, you just kind of felt like you were in the background, and you weren't seen as much. (I) think maybe it was just that. There wasn't as much connection.

While strong relationships and connections with teachers created positive memories of mathematics teachers, a lack of connection or poor relationships brought about memories of negative experiences.

Another factor that seemed to influence negative experiences with teachers was how teachers presented mathematics lessons and approached teaching. Tara and Heather described very similar experiences sitting in mathematics classrooms and being lectured to. Heather described her pre-calculus teacher in high school going through PowerPoint slide after PowerPoint slide. She recalled that "there was no actual doing of anything or learning of anything. Just being told stuff." Tara's experience was a mathematics course she took at community college. She recalled,

> It was the most awful class, probably in my college career. The teacher stood in front of the class, opened the math book, and read from the math book. There was not, like, teaching, you know? It was just verbatim out of the math book, the textbook. It was crazy. Why am I here? I can read this at my house. Yeah, and just monotone. And that was probably the worst.

Other participants recalled when the pace of the material was taught too quickly or they did not feel as though their teacher adequately explained or answered their

questions. Chloe recalled a teacher who would provide her with the answer when she was stuck or confused, rather than providing an explanation or clarifying her questions. These experiences left participants feeling frustrated, and they did not feel like they gained mathematics skills or knowledge in these courses.

Nearly all participants shared at least one memory of a mathematics teacher that was memorable to them. Some teachers left a positive impression; they left participants feeling cared for and supported as learners and students. Participants found enjoyment in mathematics class, in part because of their teachers. However, in other instances, some mathematics teachers created negative memories. The relationships were not there, the content was not presented effectively, and students did not feel supported or engaged with their learning. These findings are supported by the existing literature (Blazar & Kraft, 2017). Teachers can influence students' attitudes and behaviors positively or negatively (Blazar & Kraft, 2017). Math teachers matter. The participants' mathematics teachers influenced their feelings related to mathematics.

From Students to Teachers. Many participants discussed the transition from being a student of mathematics to becoming a teacher of mathematics. Many participants described learning mathematics when they were elementary students differently than the way mathematics was being taught today. Several participants described "aha" moments in their own understanding of mathematics concepts. Several participants described learning new strategies for approaching mathematics that differed from the memorization and focus on solving for an answer that characterized their early mathematics education. Participants discussed challenges

they experienced teaching mathematics in their classroom and during their student teaching and spoke about their hopes and intentions for teaching mathematics in the years ahead.

Participants differentiated their own experiences as students of mathematics with how they were learning to teach mathematics to students. Participants highlighted an emphasis on mathematics reasoning and "teaching students the why" during their preparation to become elementary educators. This is different from what many participants recalled from their own experiences learning mathematics in school. Meghan, Amelia, and Chloe discussed a focus on memorization over reasoning. Meghan recalled memorization and "skill and drill" as the cornerstone of her mathematics education. Similarly, Amelia remembered that memorization was one of the most challenging aspects of her mathematics experiences. Chloe described taking timed tests where she had to get through as many problems in a set amount of time. She described this as a "time crunch situation" that made her feel as though "you gotta know it." Chloe discussed how she understood fact fluency was important, but explained, "I think the way I was being asked to demonstrate it was… wasn't best for my learning."

Many participants described their own mathematics learning as a passive experience. Many spoke about teacher-directed lessons where they were expected to learn the lesson from the teacher's demonstration and then practice independently to demonstrate their ability to compute a correct answer. Mathematics was often described as a set of steps to follow with little creative flair. One participant described mathematics as feeling like "drudgery." However, not all participants felt mathematics

was boring. In fact, many participants enjoyed the subject growing up. Participants recalled many of their early mathematics experiences being primarily teacher-directed and taught from scripted mathematics programs. Naomi described a program she was taught from in elementary school:

> It was very workbook-based and that's kind of the only way I remember doing math at all, was through that workbook. And even, like, we would have our time for math and it was just, like, taught from the book.

Not many participants recalled hands-on, inquiry-based learning, nor did they discuss rich mathematics discussions. Sarah was the only participant who fondly recalled using mathematics manipulatives in her early elementary years. She described using manipulatives this way:

> So, I remember in early elementary ed., my favorite thing in the classroom was those little weight-plastic, weight bears. I don't know if anyone still uses these, but you could stack them up. It was a manipulative. You could (pause) I just couldn't wait for the chance to play with those—always.

Interestingly, in discussing their own classrooms or future classrooms, participants described mathematics learning as a much more active learning process. The mathematics learning they described was focused on engaging lessons centered on mathematics reasoning. Many participants described wanting to make mathematics "fun" for their students. Although many participants did not utilize tools and engage in hands-on mathematics learning as students, they were learning about these tools and planned to or were using them in their classrooms. Sarah strongly believed in the benefit of using tools in the classroom. Not only had she experienced her own joy in

using manipulatives as a student, but she was also learning about new tools to use in her teacher preparation program, and even seeing her own children play and engage in mathematics. She explained, “I’m raising my children and I’m noticing the art of play and learning is symbiotic.” Sarah described herself as “a big Lego mom” and had seen the value in using Legos as a mathematics tool that was both engaging for children and informative, teaching concepts such as ratios. She even enjoyed a geometry-focused lesson using origami so much that she brought origami swans home with her to share with her children.

Other participants discussed the value of using tools and hands-on learning experiences in their classrooms even though it was not something they regularly had in their own K-12 classrooms. When sharing important components of their mathematics classrooms during the focus group, Amelia and Meghan both highlighted visual tools and manipulatives. Heather recalled that she never would have considered giving students manipulatives to explore before giving direct instruction before experiencing this during her teacher preparation program. She stated:

“I think I was surprised to learn about just giving students this thing, like a manipulative of something, and not telling them what they are actually learning about yet and just seeing what they’ll do with it.” In Heather’s early mathematics years, lessons were teacher-directed, workbook based and “felt very black and white.” The way they re-experienced elementary mathematics was different from how they learned, but it was generally welcomed and accepted by participants.

Since participants had learned mathematics and experienced mathematics learning in such different ways, they had to not only shift from a student to a teacher

of mathematics but also re-evaluate their roles as a student of mathematics as well. Many participants spoke about learning new strategies and approaches to solving problems. As Chloe described, “I really had to rework the things that I had learned in order to understand it to teach it to kids.” Several participants mentioned different strategies for addition, subtraction, multiplication, and division that were different from how they learned. Amelia described recognizing misconceptions she had in her own mathematics understanding. Additionally, several participants described an increased focus on reasoning and place value understanding. Tara described learning about place value in her elementary method course:

> I didn’t learn it that way, you know? I didn’t learn you’re not carrying the one, it is one ten. Or, you know, we just didn’t learn any of that. There wasn’t a this is how you add, this is how you do it. There was no reason, you know?

Tara described fully embracing learning the strategies and found that she had even inadvertently used some of the mathematics strategies herself without knowing they were strategies. However, these newer mathematics strategies were not always initially clear to other participants. Chloe described subbing during a winter break back home and trying to help a second grader with their mathematics. She recalled, “I was looking at it and what they were doing, what they were supposed to be doing, but I was like I don’t know if I can help you because I don’t understand how it is being taught.” Similarly, Brandi recalled a learning curve during her mathematics methods course. She described, “We were like, ‘oh my god! We never learned it this way!’ which was good. I think it was good exposure. Like they are not teaching things the

same way anymore." It was a learning experience where many participants had to think about mathematics in different ways.

Several participants described "aha" moments in their own understanding of mathematics concepts and teaching. Tara described it as "enlightening," while Chloe described her mind being blown away. She recalled,

> I remember my first math class here was math for preschool-third grade and I remember the first day, my mind was blown. We were sitting there, and he put a number of objects up on the board and he was like "how many are here?" and I was like, that's easy, five. And he was like, "how do you know?" And I was like, I just counted them, that's really not that hard. And he was like, "but if a kid has never learned how to count, then how do you teach a kid to count?" Like, those kinds of things are questions that I never pondered before.

Although participants had "insider knowledge" (Pajares, 1992) of mathematics teaching and learning from their own K-12 experiences, they needed to think critically about what they knew. Simply knowing how to add, did not necessarily mean they would be successful in teaching addition to students. They had to develop mathematical knowledge for teaching.

Participants described opportunities in which they were developing their mathematical knowledge for teaching. Participants spoke favorably about many of these opportunities. Several participants described examining sample student work in their mathematics methods courses. Participants were asked to examine student work to assess the students' understanding of concepts and identify potential misconceptions in their thinking. Brandi recalled, "one of the hardest things was trying

to figure out what they did wrong. That was definitely one of the hardest things for me." Although she found it challenging, she described it as a "fun challenge" that was helpful to her as an emerging educator. Naomi also described analyzing student work and found it "fun" and "interesting." She found it valuable and described, "you can learn a lot about how to teach when you see these common errors that students are making and how to avoid those." Naomi also described a course textbook that was valuable that discussed student errors and misconceptions. She kept the book and still referred to it to identify and help address student misconceptions. In addition to seeing samples of student work in her mathematics methods courses, Chloe found it beneficial to analyze and discuss the work samples with her peers as a class. Being able to learn with and from her peers was something Chloe valued. She described,

> I remember a couple of times it would be up on the board and we'd, talk about it. Like, what do you notice? What is this student trying to do? Also, being able to work through a math scenario as a college student and being encouraged to do it in more than one way and sharing with the class. And kind of looking at what my other classmates were doing was really helpful because I got to see ways and I'd be like, 'what in the world is that?' But when they explained it, it made so much sense. But I just thought of it in a completely different way.

Chloe described that learning to anticipate her students' thinking was a valuable skill learned that she was able to use during her student teaching. During her mathematics methods course, she was taught to think about how her students may be thinking about different concepts and where the misconceptions might be. Thinking ahead this way allowed her to consider how she might proceed and address any gaps.

Participants described various opportunities to learn throughout their teacher preparation programs. Most described teacher-directed instruction in their methods courses, but many spoke of engaging in whole-class mathematics discussions with their peers. Additionally, some participants described various exposure to core practices (Grossman et al., 2009) within their teacher preparation classrooms. Participants experienced representations through student work samples and observed videos of students in the classroom. Less frequently, participants discussed opportunities for decomposition practices within their teacher preparation program. However, as discussed previously, some participants did have opportunities to analyze student work. Approximations for practice seemed to be used infrequently in participants' teacher education programs. Heather discussed teaching a lesson to her peers as they acted as students, but this type of activity did not occur regularly, and she did not receive targeted feedback or support through this exercise.

Based on what participants described, there is an opportunity to expand the use and variety of core practices within the participants' teacher education programs. With greater use of approximations of practice, such as simulations of teaching or rehearsals, participants may have the opportunity to practice their teaching skills with the guidance and support of their faculty and peers. Additionally, pre-service teachers may benefit from additional decompositions, particularly decomposing curriculum programs to better understand what is expected of curricular programs and how to modify and adapt programs to fit the needs of the students.

Participants had different views on the value of courses focused on mathematics content. Heather felt her coursework was primarily methods-focused

with little emphasis on the mathematics content she would be teaching. She described writing lesson plans and learning strategies for presenting different content; however, she wished she had had more preparation in the content knowledge. She questioned the value of learning the strategies to present mathematics without a solid foundation in the mathematics content: “How I am going to present this stuff to the student if, if I don’t actually know what it means or how to do it?” Other participants spoke of learning mathematics content during their teacher preparation programs, yet not all saw value or even relevance in this. Jon recalled feeling “confused” as to why he was learning “complex math” that he had learned in high school. He described, “the amount of complex mathematics in that class I guess was surprising to me. I didn’t feel like it was relevant for a K-6 educator.” Although he considered there may have been value in understanding beyond the grade levels he taught to understand “where it’s going,” the connection or value of what he was learning was unclear. Even for other in-service teachers, there seemed to be little focus on the significance of horizon content knowledge as an educator, for teachers to know what content comes before and after the grade(s) they teach. Although in-service teachers spoke frequently of team meetings to discuss mathematics with their grade level teams, there were no regular discussions between grade levels. Meghan discussed moving from teaching third to fourth grade and being surprised to learn how quickly the curriculum moved. She believed it would be helpful to return to third grade now knowing how the content in third grade builds the foundation for what her students would learn in fourth grade. This type of cross-grade level content understanding did not seem to be the norm among participants or within the schools they were interning or working in. Rather,

most grade level teachers seemed preoccupied with their specific grade level's content with little awareness of the curriculum and content of the grade levels before and after the grades they taught. This is an area that may benefit from further research.

In addition to developing mathematical knowledge for teaching, participants also further developed their mathematics self-efficacy and mathematics teaching self-efficacy. Sarah described her mathematical "aha moment" about her own capacity to do mathematics, something she struggled with during her K-12 experiences. She described,

> My aha moment when I realized that I could do it—that I loved math, was the class I took at [Community College]. Things that were mind-blowing to me that seemed like made sense, you could figure out, but I had no idea how you would get there. And he was teaching us things like how to quantify different levels of infinity and making that accessible.

Sarah was provided an opportunity to struggle in a supportive environment that increased her belief in her capacity to do and teach mathematics. A productive struggle was something that several other participants described as meaningful to increasing their self-efficacy.

Productive Struggle in Developing Math Self-Efficacy. Several participants spoke of challenges they experienced at one point or another in mathematics. Even for participants that saw themselves as capable mathematics students throughout school, most spoke about an experience in which they faced challenges. Participants responded to these experiences in different ways. At the time, many saw their challenges in mathematics as a reflection of their own mathematical capabilities.

As participants prepared to teach, it was valuable to gain classroom experience and fieldwork. These opportunities seemed to increase participants' confidence in their own teaching capacities the most. Participants spoke about discovering what worked and did not work with their students and in their classrooms as valuable on-the-job learning opportunities. One participant described the benefit of a productive struggle as, "you need to struggle to learn" while another participant described the value of "jumping in" to the curriculum with the guidance of a mentor teacher during teacher preparation.

Many participants discussed wishing they had more opportunities for fieldwork and time in the classroom during their teacher preparation programs. Although all participants had fieldwork as a component of their programs, this fieldwork did not always coincide with their mathematics methods coursework. Jon described a gap in being able to apply what he was learning in his methods courses:

> It was like, okay you are in this class, you are sitting. You are not really practicing the teaching of the strategies. You are practicing the strategies yourself. Practicing how you would go about teaching them and using those tools to help a student. But then you wait a semester or the next semester.

Jon went on to describe that even once he was in the classroom during his student teaching, he felt like he was mostly observing rather than having a chance to practice and engage in the strategies he had learned. He discussed wishing he had more of a chance to engage in the mathematics teaching practices in a guided way. He spoke highly of his last student teaching semester where he had a mentor teacher who would "throw" him in. This was incredibly beneficial for Jon's learning. He recalled, "That's

how I work best, is if I just get thrown into something. I gotta figure it out, I gotta have some guidance and then okay, I just got to try it." Similar to Jon, Amelia described how she had learned the most through her experiences.

Amelia was very reflective about her teaching and development as a mathematics learner and educator. She discussed her progress and spoke about failure. She once saw failure as something negative, but she came to understand that failure is a pathway to growth. She wished she had viewed failure differently during her teacher education program:

> I guess in college if you could introduce something and give people a little bit more of a chance to try it and fail, I guess. Maybe I'm just thinking about in college, like, "oh my god, failing, you're failing!" I don't think I ever fully gave into the fact that you just don't know what you are doing sometimes, like, a lot of the times. So, my mind shift has been really beneficial this year in that. And in math too, mostly math. Like, you don't have to have an answer. It's more about the process.

Considering Bandura's (1994) self-efficacy theory, performance expectations are a major component that enables individuals to develop self-efficacy. It is worthwhile to think about how teacher educators can create intentional opportunities for pre-service teachers to overcome mathematical challenges and value persistence.

The "mind shift" of viewing failure as an opportunity for growth that Amelia described may not be readily accepted by individuals, especially those who have been immersed in an achievement-oriented learning environment themselves. Therefore, opportunities to foster a growth mindset and a safe learning environment to fail can

prove advantageous in the pre-service classroom. This idea is supported in the literature, indicating that direct instruction on growth mindset concepts can productively shift pre-service teachers' beliefs about mathematics learning and teaching (Cutler, 2020). Sarah described recognizing that failure may not feel good in the moment, but it is important to recognize what you can gain from those opportunities. She said,

> I have felt the failure. I have seen people feel the failure and that is kind of like walking through the fire to get to the stuff on the other side. It doesn't feel good when it happens, but when you get through it, you are like "yes!"

Many participants, including Sarah, had learned to embrace challenges and transformed their views of failures. Existing research on mathematics failures has suggested that failure and resilience are subjective concepts that are influenced by the individual's own understanding of failure and response to failures (Lutovac, 2019). Research has suggested that some individuals view failure as an indicator that they may need to exert more effort and re-evaluate their expectations, whereas for others failure was more closely tied to one's own identity development and not viewed as something that may be readily overcome (Lutovac, 2019) This is an important area to consider, especially for individuals with lower levels of mathematics self-efficacy and mathematics teaching self-efficacy. Certain individuals who have experienced significant challenges and higher levels of mathematics anxiety may be less inclined to engage in challenging tasks or embrace failures.

Findings Summarized

This research study examined the research question: how do emerging teachers perceive the experiences and conditions, and relationships that influence their development as learners and teachers of mathematics? As a result, several insights and implications have emerged from the narratives of participants. The conceptual framework for this study considered the prior experiences and incoming beliefs of emerging educators as they entered their teacher education programs and how these influenced their capacity to learn to teach mathematics. The findings provided insights into formative experiences that molded participants' views and beliefs about mathematics learning and teaching, as well as their own mathematics self-efficacy. Societal views of mathematics as a subject, as well as an achievement-focused culture, were influential factors that shaped participants' frame of reference (Mezirow, 1997). Many emerging educators experienced mathematics learning and teaching in different ways from how mathematics is taught in classrooms today.

Although participants embraced new strategies and valued mathematics reasoning over memorization, deeply rooted habits of mind, particularly related to beliefs about mathematical abilities persisted. Additionally, participants described curriculum constraint challenges and expectations primarily associated with mathematics programs used by their schools and districts. These emerging educators did not always feel prepared to navigate the curriculum within their schools' mathematics programs. Furthermore, many participants discussed a practice disconnect between what they learned in their teacher preparation programs and the

realities of being in a classroom. They had learned best practices and strategies but did not always have a “vehicle” for these in the classroom.

Organization and Field Impacts

The outcome of this research provides insights into how emerging educators understand their own capacities to learn and teach elementary mathematics. This research considered the experiences emerging educators had with mathematics learning and teaching prior to their teacher preparation and beginning in-service teaching experiences, and how these experiences may have influenced emerging educators’ self-efficacy and ability to effectively learn to teach mathematics content. The findings of this research may be valuable for teacher educators and educational leaders to consider. To create educational systems where students are self-efficacious and successful mathematics students, educators need to be fully equipped to provide meaningful learning experiences that are equitable and accessible to all students. It is important to recognize that each emerging educator enters a teacher preparation program with a unique lens formed through their past experiences. These experiences should be recognized as the starting point from where learning begins. Teacher educators should acknowledge prior experiences and incoming beliefs of pre-service teachers and may need to provide opportunities to question and deconstruct unproductive beliefs and attitudes in the pre-service classroom. The process of becoming an elementary mathematics educator is complex and multidimensional. Examining mathematics content knowledge for teaching or mathematics self-efficacy alone may not provide an adequate understanding without careful consideration of

how an emerging educator's frame of reference shapes their viewpoints. Recommendations for practice are discussed in the following section.

Recommendations for Advocacy

There are several recommendations for practice and suggestions for further research resulting from this study. The findings of this research suggest it may be advantageous to acknowledge societal views of mathematics during teacher preparation programs. Participants in this study discussed negative societal views and rhetoric surrounding the subject of mathematics, such as a widespread dislike of mathematics and the commonly heard phrase, "I'm not a math person." Therefore, it may be useful to explore and critique these views. Specifically, it may be necessary to engage emerging educators in discussions about their own beliefs about mathematics and mathematical abilities when they enter teacher preparation programs. Teacher educators should create opportunities within coursework for teacher candidates to self-reflect on their incoming beliefs and explore common narratives about mathematics and mathematics learning. Teacher educators may ask preservice teachers to create a mathematics autobiography to learn more about the unique experiences and perspectives preservice teachers bring with them to the classroom.

Teacher educators will also need to determine if incoming pre-service teachers hold productive or unproductive beliefs about mathematics learning and teaching and consider opportunities to engage pre-service teachers in critical reflection and discourse (Mezirow, 1997). Teacher educators can create intentional opportunities to discuss negative societal rhetoric related to mathematics, and challenge ideas about abilities and achievement. These discussions should be ongoing and occur throughout

preservice teachers' programs, including during field experiences and student teaching. Teacher educators can encourage preservice teachers to think critically about existing structures that perpetuate ideas of innate intelligence and mathematical talent and provide teacher candidates with accessible ideas that foster inclusivity and equitable access to mathematics learning.

Findings also indicated emerging educators may have fixed viewpoints of their students' mathematical abilities, which may influence the instructional choices they make in the classroom. Teacher educators can introduce emerging educators to growth mindset concepts (Dweck, 2008) and ask them to critically evaluate instructional practices and decisions in the mathematics classroom. In particular, analyzing effective differentiation practices may be necessary. Emerging educators may benefit from adapting lessons to provide access and multiple entry points for learners, as well as understanding the importance of setting high expectations for learners at varying levels of understanding. Opportunities to observe effective differentiation and lesson modification, as well as time to apply these practices during field placements will further enable preservice teachers to enact high-quality mathematics teaching moving forward.

The findings of this research suggest practice disconnects exist between best practices presented within their teacher preparation programs and the realities, expectations, and curricular restraints present once educators are in their own classrooms. Therefore, it may be beneficial for teacher educators to address potential barriers and challenges with pre-service teachers. It is important to recognize and address barriers within student teaching placements and work collaboratively with

partnering school districts to promote best practices. Additionally, integrating social justice and advocacy topics within the teacher preparation curriculum may empower preservice teachers to become future change agents within their schools rather than defaulting to the status quo.

Furthermore, participants suggested they would have valued opportunities during pre-service coursework to examine scripted curriculum programs and explore how they could possibly supplement content. For example, it may be beneficial for preservice teachers to decompose a mathematics lesson from a scripted curriculum (Grossman et al., 2009) and adapt it to fit the needs of their class. Preservice teachers could be provided with a case example featuring students at various levels of understanding and be asked to adapt the script to fit the needs of various learners. Participants also commented they wish they had been exposed to and gained more accessible mathematics strategies that were readily transferable once they entered the classroom. Since several participants commented on not being able to apply strategies and approaches they learned during their teacher preparation programs once in the classroom, it may be necessary for teacher educators to provide more explicit instruction on how to incorporate concepts learned during mathematics coursework into daily practice.

Future research may include longitudinal narrative studies that track entering pre-service elementary teachers throughout their teacher education programs and into their early in-service teaching careers. This type of study would provide further insights into emerging educators' identity development and growth over time. It may be beneficial for scholars to further explore how transformational learning strategies

and practices may be utilized in the pre-service classroom to shift unproductive beliefs. If beliefs and attitudes do shift during pre-service programs, further research may also examine if productive beliefs remain once educators enter their in-service classroom teaching roles. Additionally, studies that examine emerging educators with varying degrees of mathematics self-efficacy may be beneficial, as well as research studies that explore educators from various geographical locations.

Reflective Summary

Although several insights were gained and recommendations for advocacy have been identified, there were limitations to the study that are noteworthy. If more time allowed, it would have been beneficial to extend the data collection period to track participants' stories over time. For example, it would have been interesting to interview a participant first entering their teacher preparation program, during their mathematics coursework, during their student teaching, and into their first years teaching mathematics. This would have allowed for a better understanding of their development over time and the ability for participants to share their reflections in the moment or shortly after an experience. Another initial challenge with the study involved the recruitment of participants. During the recruitment period, there were still restrictions and concerns related to the Covid-19 pandemic. Participants may have been more reluctant to participate, and it is possible the presence of external stressors related to the pandemic had a negative effect on their interest to participate. Furthermore, no participants in this study rated themselves particularly low on their self-efficacy survey. It is possible that the experiences and perspectives of pre-service and in-service teachers with low mathematics self-efficacy or low mathematics

teaching self-efficacy are vastly different from the participants in this study. It would have been beneficial to explore the narratives of participants with diverse perceptions of their mathematical capabilities.

Throughout my dissertation and doctoral journey, I have learned a great deal about my field and myself as a researcher and advocate for change. Although I never planned to engage in research during a global pandemic, I am glad that I had the experience to overcome barriers and experience some of the similar challenges my participants experienced. I hope to continue to share my work and recommendations with relevant stakeholders, particularly teacher educators. As a result of my research findings, I have an interest in further examining how transformational learning practices may be used to shift learners' counterproductive beliefs of and about mathematics learning and teaching. Additionally, I am interested in exploring student engagement in co-creating culturally and locally relevant mathematics tasks.

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Appendices

Appendix A. Recruitment to Pre-service Elementary Teacher Participants

Greetings,

My name is Stephanie Banks and I am a student in the doctoral program here at PSU. I am conducting a research study for my dissertation on how pre-service elementary teachers understand their development as math learners and teachers. As a pre-service elementary teacher your experiences and beliefs are valuable to this research study. I hope you will consider participating to share your experiences and insights.

Participation in this research study includes:

· A brief survey (up to 20 minutes)

· a focus group (60 minutes)

· an individual interview (up to 60 minutes)

If you are interested in participating in this research study, please review the informed consent form:
https://plymouthstate.co1.qualtrics.com/jfe/form/SV_2bJpB69JPfaZRhc

Thank you for your consideration and I look forward to hearing from you if you are interested in participating and/or would like to learn more about this research project.

Stephanie Banks

Doctoral Candidate

Plymouth State University

Appendix B. Recruitment to In-Service Elementary Teacher Participants

Dear Elementary Educator,

My name is Stephanie Banks and I am a doctoral student at Plymouth State University. I am conducting a research study for my dissertation on how elementary teachers understand their development as math learners and teachers. My study is looking for elementary educators in their first 1-3 years of teaching to participate. As an elementary teacher, your experiences and beliefs are valuable to this research study. I hope you will consider participating to share your experiences and insights.

Participation in this research study includes:

- A brief survey (up to 20 minutes)
- a focus group (60 minutes)
- an individual interview (up to 60 minutes)

If you are interested in participating in this research study, please review the informed consent form:
https://plymouthstate.co1.qualtrics.com/jfe/form/SV_3fKbWtuUmldcgDA

Thank you for your consideration and I look forward to hearing from you if you are interested in participating and/or would like to learn more about this research project.

If you would like additional information about this study, please contact: Stephanie Banks at smbanks@plymouth.edu or 603-479-9306.

Best,

Stephanie Banks

Doctoral Candidate

Plymouth State University

Appendix C. Informed Consent

INFORMED CONSENT FORM CONSENT TO PARTICIPATE VOLUNTARILY IN A RESEARCH INVESTIGATION PLYMOUTH STATE UNIVERSITY

INVESTIGATOR(S) NAME: Stephanie Banks, EdD candidate

STUDY TITLE: Understanding how elementary teachers view their development as math learners and teachers

PURPOSE OF THE STUDY The purpose of this research study is to explore how elementary teachers understand their development as math learners and teachers. You are being asked to be a participant in the study because as an elementary teacher in your first 1-3 years of teaching, your experience and individual perspective of math learning and teaching is believed to be valuable information that can contribute to this study.

DESCRIPTION OF THE STUDY This is a qualitative research study. Eligible individuals who choose to participate can expect to participate in a brief survey on their beliefs about math learning and teaching. Participants will also engage in a focus group with their others to discuss their experiences and perspectives of math learning and teaching. The focus group will be organized and by the Principal Investigator, Stephanie Banks. Participants can expect the focus group to run for 60-minutes. Participants will also be asked to participate in an individual interview. There will be no cost for you to participate in this research study. With participant permission, focus groups and interviews will be audio recorded. RISKS AND

DISCOMFORTS As a participant in this study, it is not anticipated that there will be more than minimal risks to you. It is possible that some participants may feel a level of discomfort in talking about their experiences. All participation is completely voluntary and participants are entitled to end their involvement in the study at any point.

BENEFITS There may be no direct benefits of participating in this study; however, the knowledge received may be of value to future teachers and their students. Your willingness to share your experiences in developing knowledge and capacities to teach mathematics can help inform teacher preparation programs and potentially in-service trainings. As a result, this may strengthen the math education for elementary students.

ALTERNATIVE PROCEDURES This research study is completely voluntary. You may choose to not participate in this study.

CONFIDENTIALITY All documents and information pertaining to this research study will be kept confidential in accordance with all applicable federal, state, and local laws and regulations. The data generated by the study may be reviewed by Plymouth State University's Institutional Review Board, which is the committee responsible for ensuring your welfare and rights as a research participant, to assure

proper conduct of the study and compliance with university regulations. If any presentations or publication result from this research, you will not be identified by name. As per federal guidelines, the information collected during your participation in this study will be kept for a minimum of three years. I plan to maintain the confidentiality of all data and records associated with your participation in this research. There are, however, rare instances when I may be required to share individually identifiable information with the following:

• Officials at Plymouth State University (PSU),

• Regulatory and oversight government agencies I also may be required by law to report certain information:

• To government and/or law enforcement officials (for example, child abuse, threatened violence against self or others, or hazing). If I believe that such a report is required, I will follow the guidance of the PSU Institutional Review Board for the Protection of Human Subjects in Research (and of the University's General Counsel) in making any such report, in order to provide as much protection for your privacy as possible while still complying with the law.

• To appropriate PSU authorities (e.g., disclosures involving Sexual Violence - which includes sexual harassment, sexual assault, unwanted sexual contact, sexual misconduct, domestic violence, relationship abuse, stalking [including cyber-stalking] and dating violence - must be reported to the PSU Title IX Coordinator or PSU Police). Further, any communication via the internet poses minimal risk of a breach of confidentiality. While I plan to maintain confidentiality of your responses, other focus group participants may repeat responses outside the focus group setting. To help protect the confidentiality of your information, all data collected from the study will be coded to protect your name and identifying information. No names or identifying information will be used when discussing or reporting on this research. I will report the data using pseudonyms. During focus groups, anonymity will be preserved by refraining from using actual names. Each participant will be given a tent card with an alphabetic letter to refer to themselves and others. Once audio recordings are transcribed and coded, they will be destroyed. Data will be stored on the researcher's password protected personal computer and will not be accessed on a public or shared device. Data will be kept for 3 years and then destroyed. The results may be used in reports, presentations, and publications.

TERMINATION OF PARTICIPATION You may choose to withdraw from this study at any time and for any reason. If you choose to drop out of the study, you may contact the investigator and your research records will be destroyed. If you have not completed a field experience or completed math requirements of your program, you may be excluded from this study.

COMPENSATION You will not receive payment for being in this study. Participation in this study is strictly voluntary. There will be no cost to you for participating in this research.

INJURY COMPENSATION Neither Plymouth State University nor any government or other agency funding this research project will provide special services, free care, or

compensation for any injuries resulting from this research. The treatment for such injuries will be at your expense and/or paid through your medical plan.

QUESTIONS If you have further questions about this study, you may contact Stephanie Banks (Principal Investigator) or Annette Holba (Faculty Chair) at smbanks@plymouth.edu or aholba@plymouth.edu. If you have any questions about the rights of research participants, you may call the Chairperson of the Plymouth State University's Institutional Review Board at 603-535-2915 (Valid until July 1, 2024).

VOLUNTARY PARTICIPATION You understand that your participation in this study is entirely voluntary, and that refusal to participate will involve no penalty or loss of benefits. You are free to withdraw or refuse consent, or to discontinue your participation in this study at any time without penalty or consequence. You voluntarily give your consent to participate in this research study. You will be given a copy of this consent form:.

I, the undersigned, certify that to the best of my knowledge, the subject signing this consent form has had the study fully and carefully explained by me and have been given an opportunity to ask any questions regarding the nature, risks, and benefits of participation in this research study.

Investigator's Name
Stephanie Banks

Date: 4/9/2022

Plymouth State University's IRB has approved the solicitation of participants for the study until January 13, 2023

Appendix D. Focus Group Protocol

Welcome & Informed Consent Review:

Researcher: Thank you all for coming today. I appreciate your willingness to participate in this research study and take part in today's focus group. I want to begin today by reviewing the informed consent form you completed prior to today's session and answer any questions or concerns you may have.

Review informed consent form

Researcher: Does anyone have any questions or concerns?
Answer/address any questions or concerns

Purpose/Overview:

The purpose of today's focus group is to encourage you all to talk to one another. I will pose questions and topics, but this is to promote discussion. Please do not feel the need to respond to me directly, but rather talk with one another. I am here as a facilitator and observer. The purpose of this research study is to understand your experiences and viewpoints as emerging elementary teachers who are learners and teachers of mathematics. There are no right or wrong answers and what is said here today should remain confidential and private. Nothing said will impact your grade or program completion. I will not be sharing what you, as individuals, say with your professors, field supervisors, or mentor teachers. Your openness and willingness to share thoughts, ideas, and experiences will both enrich this discussion and contribute to the research project.

Ground Rules

Researcher: To make our session most productive today, I want to establish a few simple guidelines.

1) Let us have only one person speaking at a time. This is going to help make sure I can hear all the important things you have to say, in addition to having the chance to hear what each of your peers has to say.
2) Please silence your cell phone or put it out of sight. Please try to be present for our 60-minutes today and limit distractions.
3) As I mentioned in our consent form, confidentiality and privacy are of utmost importance. I ask all of you to uphold this amongst yourselves. Please do not share what is said with others outside of this group. I am asking you all to share your experiences and this requires a level of vulnerability to do so. We can all feel more comfortable and able to share and provide input when we know we are in a safe and confidential space.

Researcher: Do these sound like reasonable guidelines that you are able to follow? Any other questions or concerns?

Researcher: I would like to record today's session so that I do not miss any of the important comments and points you make. After I have transcribed and coded the information, I will delete the recording. Does anyone have any concerns about being recorded?

Focus Group Questions

Opening question (everyone answers): In a single word, what comes to mind when I say "math".

Beliefs about Mathematics

1) In your own words, what is mathematics?
    a. *Probe*: Imagine meeting someone with no context or background with mathematics. How would you describe it?
    b. *Probe:* What value does mathematics have? What role does it have in our lives/society?
    c. *Probe*: What is mathematics not?
        i. *Probe:* Misconceptions?
        ii. *Probe:* Differences from other subject areas?

Defining Early Experiences

2) What has your experiences been as a student of mathematics (K-12)?
    a. *Probe* : K-12: What are your earliest memories of mathematics?
    b. *Probe:* Positive memories?
    c. *Probe:* Negative memories?

Transition from Learner to Teacher of Math

3) Let's shift gears and talk about more recent experiences in your teacher preparation program. How would you describe the transition from student to future educator?
    a. *Probe:* How has that transition been like with mathematics specifically?
    b. *Probe:* Compare your experience learning elementary math as a K-12 student and as a pre-service teacher.

Evidence of Factors that Influence Self-efficacy

4) Talk to me about math failures. Has anyone felt unsuccessful before when engaging with mathematics?
    a. *Probe:* Describe what happened and how you felt?
    b. *Probe*: How have you overcome math "failures" and found successes?
        i. *For the researcher: Mastery experiences, verbal persuasion, vicarious experiences, psychological arousal.*
    c. *Probe:* Tell me about any lingering concerns and fears about failure you may still have?

Opportunities within Teacher Preparation Program to Influence Self-efficacy

5) How would you describe the experiences and opportunities you have had as part of your teacher preparation program to develop your **math knowledge?**
    a. Talk to me about your coursework in mathematics specifically.
    b. Talk to me about your field experiences-what responsibilities and opportunities have you had to teach and work with students in mathematics?
        i. *Probe:* What has been beneficial?
        ii. *Probe:* What has not been helpful?
        iii. *Probe:* What is missing?
        iv. *Probe:* What do you wish you had more of?
6) How would you describe the experiences and opportunities you have had as part of your teacher preparation program that have prepared you to **teach mathematics effectively**? (best practices, NCTM principles, meeting CCSS-M)
    a. *Probe:* What has been beneficial?
    b. *Probe:* What has not been helpful
    c. *Probe:* What is missing
    d. *Probe:* What do you wish you had more of
7) Is there anything we have not discussed today about teaching, learning, and experiences with mathematics that we should discuss?

<u>Concluding Remarks</u>

Researcher: We are coming to the end of our focus group session today. Thank you all for attending and participating in this research.

Provide contact information for follow-up/additional information

Appendix E. Interview Protocol

Review Informed Consent:

Researcher: Thank you for taking time out of your day to speak with me and participate in this research study. Before we begin, I want to review the consent form you completed and answer any questions you may have.

Review informed consent form.

Researcher: Do you have any questions or concerns before we continue?

Introduction:

Researcher: You have been asked to participate in this research because your experience as an elementary pre-service teacher, who is preparing and learning to teach elementary mathematics, is valuable to the research. The purpose of this study is to understand your perspectives and experiences and I encourage you to be open and honest. There are no right or wrong answers. Everything you have to say is valid. I will make every effort to ensure confidentiality and privacy, as discussed in the consent form. What is said here will not impact your grade or teacher preparation program completion. I will not be speaking with your university, professors, or mentor teacher about what you say, specifically. When the information is reported out pseudonyms will be used.

Researcher: I would like to record our interview today so that I do not miss anything that you say. Would it be okay with you if I record? If at any time during the interview you would like me to pause or stop the recording, please let me know and I will stop it.

Start recording with permission from participants.

Interview Questions-Pre-service Teacher Interview

Questions 1-6 will be informed by/follow-up from focus-groups

1) How would you describe your feelings about mathematics as a subject?
2) Can you tell me about your past experiences being a student of mathematics in a K-12 setting?
    a. Which moments stick out to you that you think formed or changed your beliefs about yourself as a math learner?
3) When you entered your teacher preparation program, how might you have described your readiness and ability to teach elementary math?
    a. Tell me about your experiences learning elementary mathematics and learning to teach that mathematics from your teacher preparation program.

   b. Which experiences, assignments, and events stick out to you that you think formed or changed your beliefs, either as a math learner or as a future teacher of math?

4) At this moment, how do you describe *your* math capabilities?
   a) How confident do you feel in your knowledge of elementary math concepts? Math procedures? What about concept behind a standard procedure?
5) How would you describe an effective elementary math teacher and what do you think math learning should look like in the elementary classroom?
   a. Follow-up: how do you see yourself embodying those characteristics/elements?
6) How well do you feel your experiences gained through your teacher preparation program have prepared you to teach elementary mathematics?
   a. Follow up—is there anything/what would you add, omit, change about the teacher education program or your experience so far?
   b. What experiences or events outside of your teacher preparation program do you feel have prepared you for teaching elementary mathematics?

Questions 7-11 all addressed during interview

7) If you were to start tomorrow in a new role as a full-time classroom teacher, talk about your readiness and comfort to teach and assess your students in mathematics?
   a. *Probe:* How does your readiness and comfort change based on the grade level of the position? 1st grade vs. 5th grade?
8) Describe your readiness and ability to lead a rich math discussion.
   a. Follow up-what teaching skills and content knowledge do you think are needed to successfully do this?
   b. How would you feel if a student asked a question you were not sure the answer of you were not understanding their thought process?
9) Describe your readiness and ability to elicit and understand students' math thinking.
   a. How confident are you in examining student work to identify depth of understanding or possible conceptual or procedural misconceptions? What about identifying depth of understanding or misconceptions during classroom discussions?
10) *Give a complex math* problem (display or hand out)
   a. How do you feel about your ability to solve this problem?
      i. (*For the researcher:*)underlying concepts, lead a discussion, answer student questions, evaluate depth of understanding/identify student misconceptions related to this problem (tell me more)
   b. How do you feel about your ability to explain the concepts and procedures behind what you would do to solve this?
   c. How do you feel about explaining the concepts and procedures to students?

d. How do you manage students at various levels of understanding?
    i. Is this problem too difficult for some students? Too easy for others?
    ii. How much time do you allow students to work through this?
    iii. What do you do when they are struggling? How do you know when to intervene?

11) Is there anything you thought I would ask you that I did not or anything else you would like to tell me?

Concluding Remarks

Researcher: Thank you for taking time to speak with me today to share your experiences and perspective. I will be reaching out to you to send you a transcript of our interview today. I would ask that you kindly review the transcript to ensure I have captured today's session with accuracy. If you notice any discrepancies, please let me know. If you think of any other information or thoughts you would like to add after today, please feel free to reach out to me. I will leave you with my contact information. If you would be willing to participate in an individual interview, please leave me your contact information.

Provide contact information for follow-up/additional information